X100/301

NATIONAL QUALIFICATIONS 2006

FRIDAY, 19 MAY
9.00 AM – 10.10 AM

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

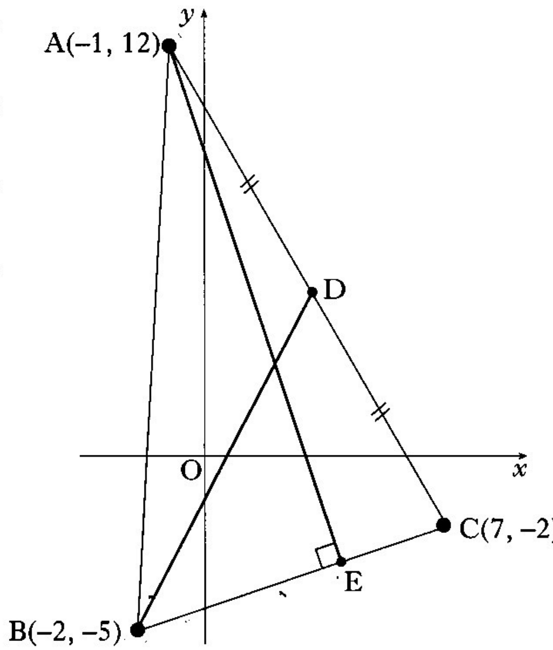
Marks

1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

   (*a*) Find the equation of the median BD. **3**

   (*b*) Find the equation of the altitude AE. **3**

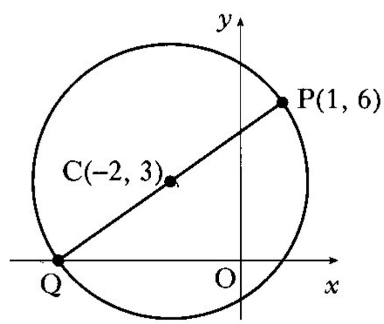
   (*c*) Find the coordinates of the point of intersection of BD and AE. **3**



2. A circle has centre C(–2, 3) and passes through P(1, 6).

   (*a*) Find the equation of the circle. **2**

   (*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

   (*a*) Find expressions for:

   (i) $f(g(x))$;

   (ii) $g(f(x))$. **3**

   (*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$. **2**

[Turn over

Marks

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

(a) State why this sequence has a limit. **1**

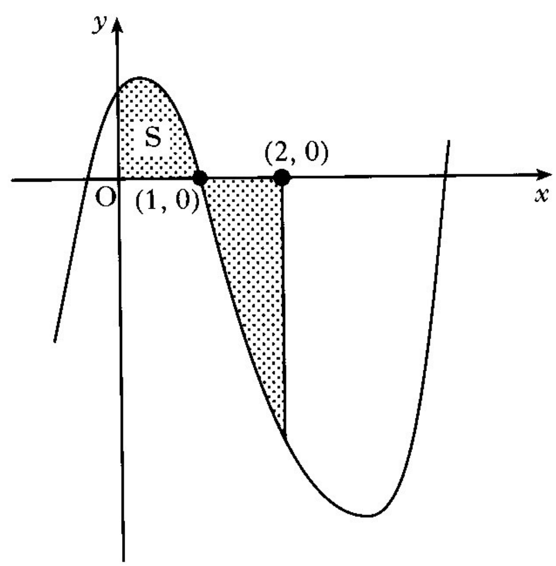
(b) Find this limit. **2**

5. A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.



(a) Calculate the shaded area labelled S. **4**

(b) Hence find the total shaded area. **3**

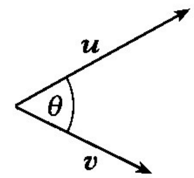
7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \leq x \leq 360$. **4**

8. (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

Marks

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



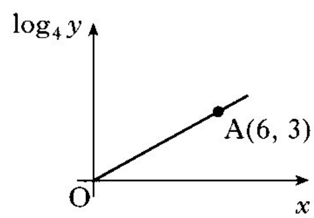
(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2**

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5**

(*c*) Deduce the only possible value of k. **1**

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. **4**



[*END OF QUESTION PAPER*]

X100/303

NATIONAL QUALIFICATIONS 2006

FRIDAY, 19 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

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Marks

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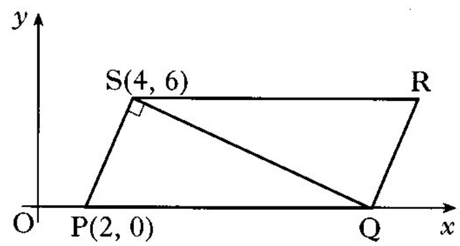
The diagonal QS is perpendicular to the side PS.

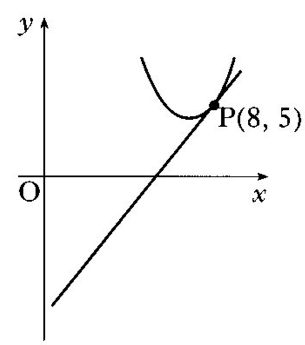


(a) Show that the equation of QS is $x + 3y = 22$. **4**

(b) Hence find the coordinates of Q and R. **2**

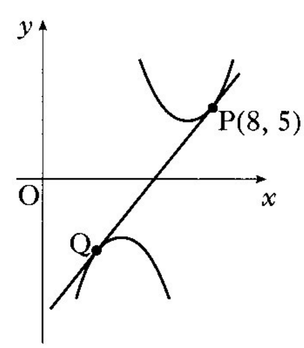
2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).



(a) Find the equation of this tangent. **4**

(b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.

Determine the radius of the larger circle. **5**

Marks

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

6. P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$.

(*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

(*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

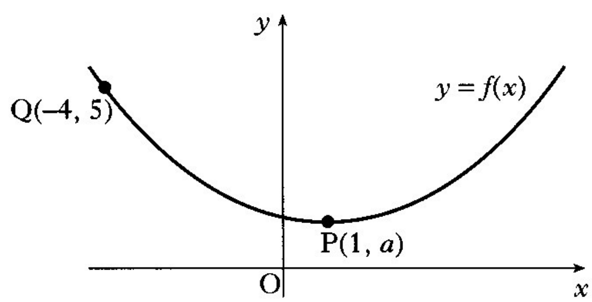
(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

7. The diagram shows the graph of a function $y = f(x)$.

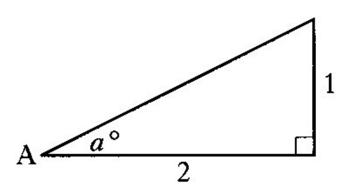
Copy the diagram and on it sketch the graphs of:

(*a*) $y = f(x - 4)$; **2**

(*b*) $y = 2 + f(x - 4)$. **2**



8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(*a*) Find the exact values of:

(i) $\sin a°$;

(ii) $\sin 2a°$. **4**

(*b*) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$. **4**

9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. **4**

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. **4**

(*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

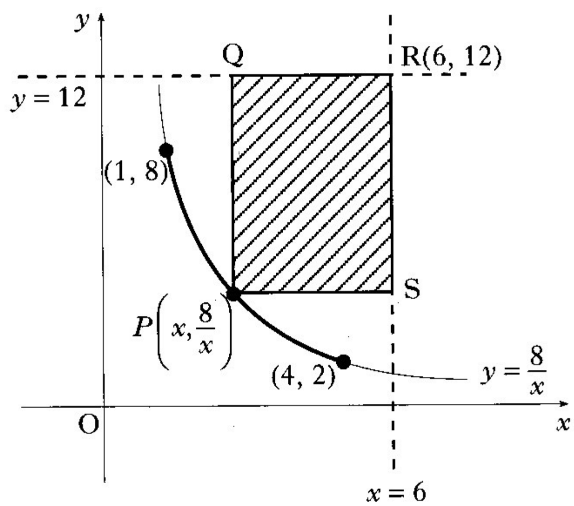
The formula $A(t) = A_0 e^{-0\cdot000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true? **5**

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

[END OF QUESTION PAPER]