X100/301

NATIONAL
QUALIFICATIONS
2002

MONDAY, 27 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**
2 Full credit will be given only where the solution contains appropriate working.
3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

2. The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1. Find the coordinates of Q. **3**

3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(a) Find expressions for:

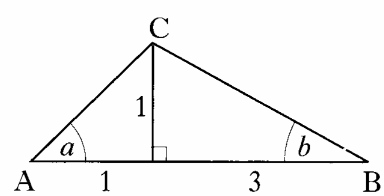
(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(b) Solve $2f(g(x)) = g(f(x))$ for $0 \le x \le 360$. **5**

4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

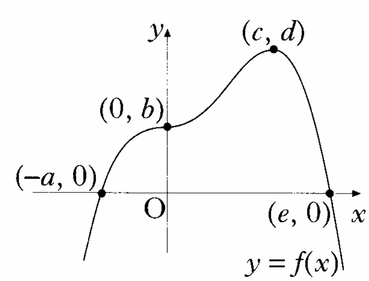
5. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. **4**



6. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**



[Turn over for Questions 7 to 11 on *Page four*

Marks

7. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

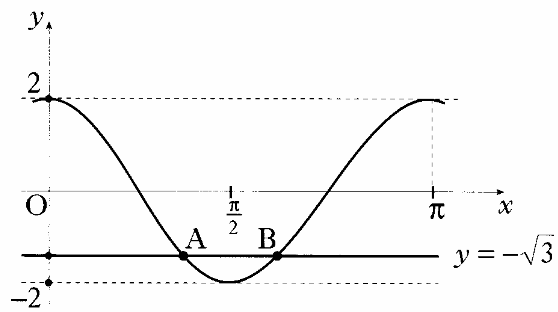
(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

8. The diagram shows the graph of a cosine function from 0 to π.



(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. **3**

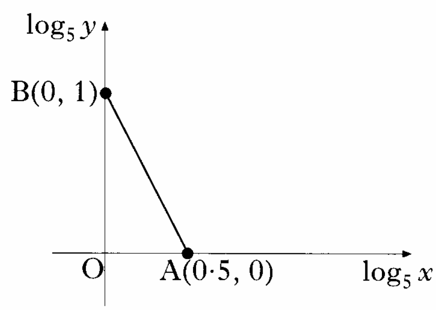
9. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

10. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. **2**

(*b*) Hence write down $\displaystyle\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}}\,dx$. **1**

11. The graph illustrates the law $y = kx^n$. If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. **4**



[*END OF QUESTION PAPER*]

X100/303

NATIONAL
QUALIFICATIONS
2002

MONDAY, 27 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

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LIB X100/303 6/27070

ALL questions should be attempted.

Marks

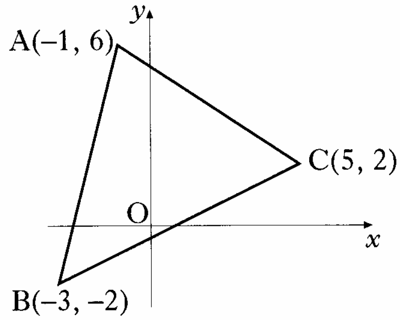
1. Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2).

   Find

   (*a*) the equation of the line p, the median from C of triangle ABC. **3**

   (*b*) the equation of the line q, the perpendicular bisector of BC. **4**

   (*c*) the coordinates of the point of intersection of the lines p and q. **1**



2. The diagram shows a square-based pyramid of height 8 units.

   Square OABC has a side length of 6 units.

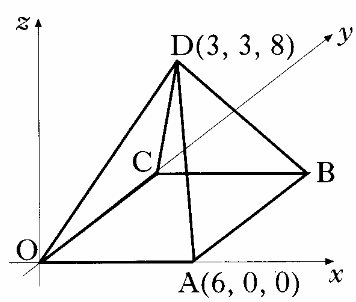
   The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

   C lies on the y-axis.

   (*a*) Write down the coordinates of B. **1**

   (*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

   (*c*) Calculate the size of angle ADB. **4**

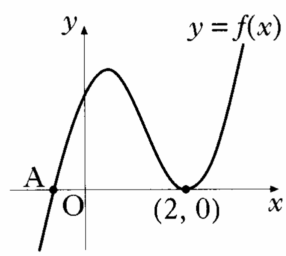


3. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

   (*a*) Find the x-coordinate of the maximum turning point. **5**

   (*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

   (*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**



[Turn over

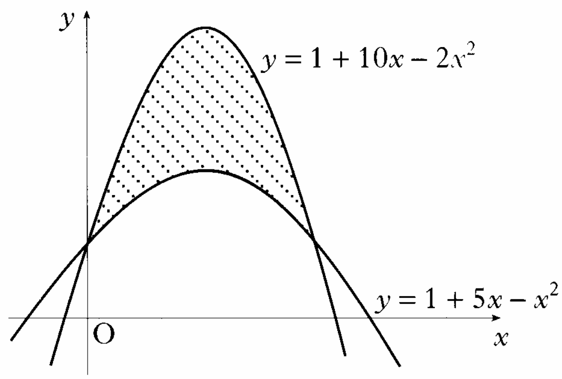
Marks

4. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

   (*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? **3**

   (*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? **3**

5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**



6. Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

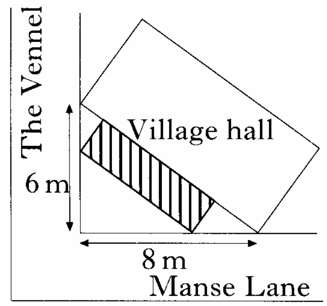
7. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x - 2) + 1$ intersects the x-axis. **3**

8. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

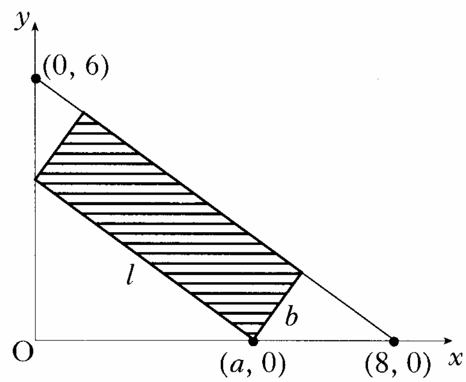
9. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Marks

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, $A\,\text{m}^2$, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

[*END OF QUESTION PAPER*]