1) $y = mx + c$

$m = -\frac{2}{3}$

$y = -\frac{2}{3}x + c$

$(2, -1)$

$-1 = -\frac{4}{3} + c$

$c = \frac{1}{3}$

$\therefore y = -\frac{2}{3}x + \frac{1}{3}$

$3y + 2x = 1$

2) $x^2 - 5x + (k+6) = 0$

equal roots when discriminant = 0 :

i.e when:

$25 - 4(k+6) = 0$

$\Rightarrow k = \frac{1}{4}$

3)(a) $A(-8,-10,-2)$, $B(-2,-1,1)$, $C(6,11,5)$

$\vec{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\quad \vec{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = 4\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$\therefore$ as $\vec{AB} = k\vec{BC}$ and both share B as a common point, ABC is a straight line.

(b) $\vec{OB} = \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix}$

$\vec{AB} \perp \vec{OB} \Leftrightarrow \vec{AB} \cdot \vec{OB} = 0$.

$\vec{AB} \cdot \vec{OB} = -18 + 27 - 9$

$= 0$ ∎

4) $f(x) = x^2 + 2x - 8$

$= (x+1)^2 - 8 - 1$

$= (x+1)^2 - 9$.

5) (a) $\sin 2x - \cos x = 0 \qquad 0 \le x \le 180$

$\Rightarrow 2\sin x \cos x - \cos x = 0$

$\Rightarrow \cos x \,(2\sin x - 1) = 0$

$\Rightarrow \cos x = 0$ or $\sin x = \frac{1}{2}$

$\Rightarrow x = 90^\circ$ or $x = 30^\circ, 150^\circ$.

(b) $P(150, -\frac{\sqrt{3}}{2})$.

6) $P(x) = 12x^3 - x^4 \qquad 0 \le x \le 12$

Stationary point when $P'(x) = 0$

i.e when $36x^2 - 4x^3 = 0$.

$\Rightarrow 4x^2(9 - x) = 0$

$\Rightarrow x = 0$ or $x = 9$

$\therefore$ Max when $x = 9$ (from diagram)

7) $f(x) = \sin x$

$g(x) = \cos x$

$h(x) = x + \pi/4$

(a)(i) $f(h(x)) = f(x + \pi/4)$

$= \sin(x + \pi/4)$

(ii) $g(h(x)) = g(x + \pi/4)$

$= \cos(x + \pi/4)$

(b) See over...

8) $4\log_x 6 - 2\log_x 4 = 1$

$\Rightarrow \log_x\left(\frac{6^4}{4^2}\right) = 1$

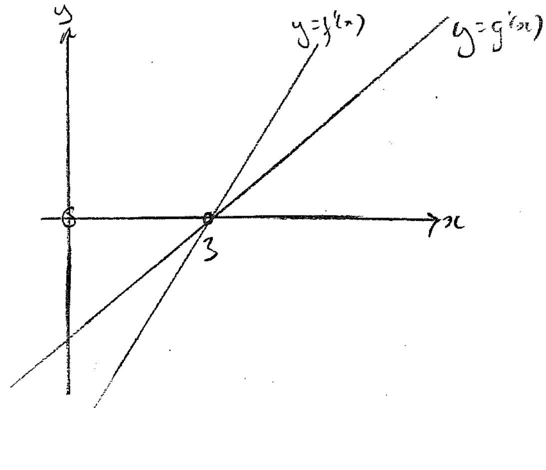
$\Rightarrow \log_x\left(\frac{6^4}{2^4}\right) = 1$

$\Rightarrow \log_x 3^4 = 1$

$\Rightarrow x = 3^4$

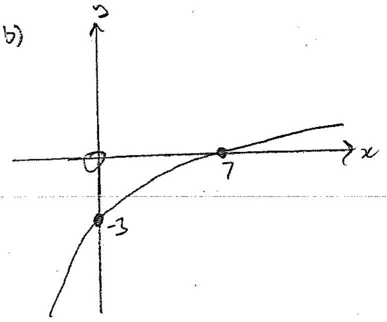
$\Rightarrow x = 81$

9)



10) (a) $a = 1$
$b = 3$

(b)



11) (a) (i) $\text{radius}_P = \sqrt{(-4)^2 + (-5)^2 - 9}$
$= \sqrt{32}$
$= 4\sqrt{2}$

(ii) $\text{Centre}_P : (4, 5)$
$\text{Centre}_Q : (-2, -1)$

Distance between $(4,5)$ & $(-2,-1)$ is:

$\sqrt{6^2 + 6^2}$
$= \sqrt{72}$
$= 6\sqrt{2}$
$= 4\sqrt{2} + 2\sqrt{2}$
$= \text{radius}_P + \text{radius}_Q$
$\Rightarrow$ two circles touch.

(b) Let centre of Q be point C and $(-4, 1)$ be point A.

Then, $M_{CA} = \dfrac{(-1) - 1}{(-2) - (-4)}$

$= -1$

$\therefore M_{\text{tangent}} = 1$

$y = mx + c$

$y = x + c$

$(-4, 1)$

$1 = -4 + c$

$c = 5$

$\therefore y = x + 5$

(c) $x^2 + y^2 - 8x - 10y + 9 = 0$
intersects $y = x + 5$ when:

$x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$
$\Rightarrow x^2 + x^2 + 10x + 25 - 8x - 10x - 50 + 9 = 0$
$\Rightarrow 2x^2 - 8x - 16 = 0$
$\Rightarrow x = \dfrac{8 \pm \sqrt{64 + 128}}{4}$

$\Rightarrow x = 2 \pm 2\sqrt{3}$

$\Rightarrow x = a \pm b\sqrt{3}$ with $a = b = 2$.

7) (b) (i) $f(h(x)) = f\left(x + \frac{\pi}{4}\right)$
$= \sin\left(x + \frac{\pi}{4}\right)$
$= \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$
$= \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$

(ii) $g(h(x)) = \cos\left(x + \frac{\pi}{4}\right)$
$= \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}$
$= \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$

$f(h(x)) - g(h(x)) = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x\right) - \left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x\right) = 1$

$\Rightarrow \frac{2}{\sqrt{2}} \sin x = 1$

$\Rightarrow \sin x = \frac{\sqrt{2}}{2}$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$

S	A
T	C

2001 : Paper 2

1) (a)

$$\begin{array}{r|rrrr} -2 & 2 & 1 & k & 2 \\ & & -4 & 6 & -12-2k \\ \hline & 2 & -3 & 6+k & -10-2k \end{array}$$

$x+2$ a factor $\Rightarrow -10-2k = 0$

$\Rightarrow k = -5$

(b) $2x^3 + x^2 - 5x + 2 = 0$

$\Rightarrow (x+2)(2x^2 - 3x + 1) = 0$

$\Rightarrow (x+2)(2x-1)(x-1) = 0$

$\Rightarrow x = -2$ or $x = \frac{1}{2}$ or $x = 1$

2) $y = x - 16x^{-1/2} \quad (x > 0)$

$\Rightarrow \frac{dy}{dx} = 1 + 8x^{-3/2}$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{8}{(\sqrt{x})^3}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=4} = 1 + 1$

$= 2$

$y = mx + c$

$\therefore y = 2x + c$

$(4, -4)$

$-4 = 8 + c$

$c = -12$

$\therefore y = 2x - 12$

3) (a) $u_{n+1} = 1.015u_n - 300$, $u_0 = 2500$

(b) $u_1 = 2237.5$

$u_2 = 1971.0625$

$u_3 = 1700.6284$

$u_4 = 1426.1379$

$u_5 = 1147.5299$

$u_6 = 864.7429$

$u_7 = 577.7140$

$u_8 = 286.38$

interest added on last month

$\therefore$ last payment of 286.38×1.1015

$= £290.68$ on 1^{st} Dec.

4) $\cos(A\hat{B}C) = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|}$

$\vec{BA} = \underline{a} - \underline{b}$ $\qquad$ $\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix}6\\0\\7\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix}$ $\qquad$ $= \begin{pmatrix}4\\5\\0\end{pmatrix} - \begin{pmatrix}0\\5\\6\end{pmatrix}$

$= \begin{pmatrix}6\\-5\\1\end{pmatrix}$ $\qquad$ $= \begin{pmatrix}4\\0\\-6\end{pmatrix}$

$\therefore \vec{BA} \cdot \vec{BC} = 24 - 6$

$= 18$

$|\vec{BA}| = \sqrt{36 + 25 + 1}$ $\qquad$ $|\vec{BC}| = \sqrt{16 + 36}$

$= \sqrt{62}$ $\qquad$ $= \sqrt{52}$

$\therefore \cos(A\hat{B}C) = \frac{18}{\sqrt{62}\sqrt{52}}$

$\Rightarrow A\hat{B}C = 71.5^\circ$ (3SF)

5) $8\cos x - 6\sin x = k\cos(x + a) \quad k > 0, 0 < a < 360.$

$k\cos(x+a) = k\cos x\cos a - k\sin x\sin a$

$\therefore \quad k\cos a = 8$

$k\sin a = 6$

$\Rightarrow k = \sqrt{64 + 36}$

$\Rightarrow k = 10 \quad (k > 0).$

also $\tan a = \frac{6}{8}$

$\Rightarrow a = 36.9^\circ$ (3SF)

$\therefore 8\cos x - 6\sin x = 10\cos(x + 36.9)$

6) $\int \frac{(x^2-2)(x^2+2)}{x^2} dx \quad x \neq 0$

$= \int \frac{x^4 - 4}{x^2} dx$

$= \int (x^2 - 4x^{-2}) dx$

$= \frac{1}{3}x^3 + \frac{4}{x} + c$

7) (a) $l_1 : x = 7$

(b) $m_{AC} = \frac{6-2}{8-2}$

$= \frac{2}{3}$

$\therefore m_{l_2} = -\frac{3}{2}$

$y = mx + c$

$y = -\frac{3}{2}x + c$

$(5, 4)$

$4 = -\frac{15}{2} + c$

$c = \frac{23}{2}$

$\therefore l_2 : y = -\frac{3}{2}x + \frac{23}{2}$

i.e. $l_2 : 2y + 3x = 23$

(c) l_1 intersects l_2 when:

$2y + 21 = 23$

$\Rightarrow y = 1$

$\therefore$ point of intersection is:

$(7, 1)$

(d) Centre of circle is $D(7, 1)$

radius $= \sqrt{1^2 + 5^2}$

$= \sqrt{26}$

$\therefore$ eqn of circle is:

$(x-7)^2 + (y-1)^2 = 26$

8) $y = (x+1)(x-1)(x-3)$

$y = 5x - 5$

Area $= 2\int_1^4 \left((5x-5) - (x+1)(x-1)(x-3)\right) dx$

$= 2\int_1^4 \left((5x-5) - (x^2-1)(x-3)\right) dx$

$= 2\int_1^4 \left((5x-5) - (x^3 - 3x^2 - x + 3)\right) dx$

$= 2\int_1^4 (-x^3 + 3x^2 + 6x - 8)\, dx$

$= 2\left[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x\right]_1^4$

$= 2\left((-64 + 64 + 48 - 32) - (-\frac{1}{4} + 1 + 3 - 8)\right)$

$= 40\frac{1}{2}$ units^2

9) $A = A_0 e^{kt}$

$2A_0 = A_0 e^{1.5k}$

$\Rightarrow e^{1.5k} = 2$

$\Rightarrow 1.5k = \ln 2$

$\Rightarrow k = \frac{2}{3}\ln 2$

$\Rightarrow k = 0.46$ (2DP)

10) $\frac{dy}{dx} = 3\sin(2x)$

$\Rightarrow y = -\frac{3}{2}\cos(2x) + c$

$(\frac{5\pi}{12}, \sqrt{3})$

$\sqrt{3} = -\frac{3}{2}\cos(\frac{5\pi}{6}) + c$

$\Rightarrow \sqrt{3} = \frac{3\sqrt{3}}{4} + c$

$\Rightarrow c = \frac{\sqrt{3}}{4}$

$\therefore y = -\frac{3}{2}\cos(2x) + \frac{\sqrt{3}}{4}$

11) (a) General eqn of parabola is:

$y = ax^2 + bx + c.$

$(-1, 0): 0 = a - b + c$ — ①

$(0, p): p = c$ — ②

$(p, 0): 0 = ap^2 + bp + c$ — ③

② in ③: $ap^2 + bp + p = 0$

$\Rightarrow ap + b + 1 = 0 \quad (p \neq 0)$

$\Rightarrow ap + b = -1$ — ④

① is: $a - b = -p$

④ + ①: $a(p+1) = -(p+1)$

$\Rightarrow a = -1$

in ①: $b = p - 1$

$\therefore$ parabola is: $y = -x^2 + (p-1)x + p$

(b) $y = x + p$ meets parabola when:

$-x^2 + (p-1)x + p = x + p$

$\Rightarrow -x^2 + (p-2)x = 0$

$\Rightarrow x^2 + (2-p)x = 0$

$\Rightarrow x(x + 2 - p) = 0$

$\Rightarrow$ tangent if $p = 2$ (i.e. one sln, $x=0$)