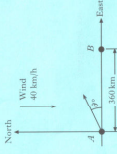


8. The acceleration of a particle travelling in a straight line is given by $\frac{1}{1+x^2} \text{ m s}^{-2}$, where x is the time in seconds since the particle started moving. Given that the velocity is zero when $x=1$, find the velocity when $x=3$. 4

9. An electronic device will operate correctly provided at least four of the five chips in the device are working. The probability that a chip is defective is $\frac{1}{100}$. Find the probability that the device operates correctly. 3
The device is redesigned and the original five chips are replaced by three new chips. The probability that the device operates correctly is now $\frac{1}{10}$. Does the device operate correctly? The device now requires all three of the new chips to be non-defective. Is the new design more reliable than the original? Justify your answer. 2

10. Express in partial fractions $\frac{11x-26}{x^3+x^2-2}$. 3
Hence obtain $\int \frac{11x-26}{x^3+x^2-2} dx$. 3

11. A helicopter sets off from an airfield A to fly to an oil rig B located 360 kilometres due East of A , as shown in the diagram. Its still air, no-wind, ground speed is 40 km/h. The helicopter is blown off course by a wind from the North. To compensate for this, the pilot flies the helicopter in the direction θ° North of East. 3



- (a) Find the resultant velocity of the helicopter in terms of θ , k and j , where i and j denote unit vectors in the directions of East and North respectively. 3
(b) Deduce that in order to fly directly towards B , $\theta = \sin^{-1} \frac{3}{4}$. 3
(c) If the airspeed, you should assume that the helicopter's altitude and speed are constant for the entire duration of the flight. 2

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Page four

SECTION I

1. (a) Differentiate $f(x) = e^{x^2}$. 2
(b) Differentiate $g(x) = \ln \sqrt{x^2+3}$. 2
Hence find $\int \frac{8x}{x^2+3} dx$. 2

2. The profits, in £1000s, made by the Tangmere Mobile Phones Company in its first 25 months of trading were recorded and are given in the table below. 4
Month 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Profit 4 5 6 10 12 15 15 15 15 13 21 26 28 31 33 33 33 32 32 30 52 49 47 42 41 41 48 49 51
Month 14 15 16 17 18 19 20 21 22 23 24 25
Profit 33 32 30 52 49 47 42 41 41 48 49 51 51
(a) Construct a stem-and-leaf diagram for this data. 2
(b) State the values of the median and the quartiles. 2

3. Let A be the matrix

$$A = \begin{pmatrix} -1 & 6 & -3 \\ -1 & -1 & 7 & -3 \\ -1 & 12 & -8 \end{pmatrix}$$

4. Show that $A^2 + A^T - A = I$ where I denotes the 3×3 identity matrix. 4

5. Find the complex number $z = a + bi$ where I denotes the modulus and argument of z . 3

6. Calculate $\frac{d}{dx} \int_0^x \cos t dt$ where π denotes the complex conjugate of x . 2
Use de Moivre's theorem to evaluate $e^{i\pi}$. 2

7. Use the composite trapezium rule with four strips to estimate $\int_0^1 \frac{5-x}{2+3x^2} dx$. 4
State your answer to three decimal places.

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Page two

SECTION II

Attempt ALL questions in this section.

12. The function f is defined by $f(x) = e^{\cos x}$, $0 \leq x \leq \frac{\pi}{2}$. 4

- (a) Find the stationary point of f and determine its nature. 5
Sketch the graph of f showing clearly where the graph meets the x -axis and the y -axis. 5

- (b) By integrating by parts twice, show that $\int_0^{\frac{\pi}{2}} f(x) dx = \frac{1}{2} e^{\cos x} (\cos x + \sin x) + C$ 4

- where C denotes the constant of integration. 4
Use this result to calculate the area bounded by the graph of f and the coordinate axes. 2

13. A car manufacturer is planning to use production patterns. Based on estimates of time, cost and labour, he obtains a set of three equations for the numbers x, y, z of three new types of car. These equations are $x + 2y + 3z = 20$, $3x + y + 4z = 105$, $5x + 2y + z = 60$. 5

- where the integer k is a parameter such that $0 < k < 10$. 5
(a) Use Gaussian elimination to find an expression for x in terms of k . 2
(b) Given that x must be a positive integer, what are the possible values for k ? 2

- (c) Find the corresponding values of x and y for each value of k . 2
14. By writing $(k+1)^2 - k^2 = 2k+1$, 5

- show that $\sum_{k=1}^n k^2 = \frac{1}{6} n(n+1)(2n+1)$. 5
Deduce that $\sum_{k=1}^n k^2 = \frac{1}{6} n(n+1)(2n+1)$. 2

- Hence obtain the sum of the squares of all the even integers between 99 and 201. 2

[Turn over for Question 15 on page six]

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Page five

Marks

6. The function g is defined by $g(x) = \tan^{-1} x$, $x \in \mathbb{R}$. 5
Verify that the second derivative of g is given by $g''(x) = \frac{-2x}{(1+x^2)^3}$. 1

- where C is a constant. State the value of C . 5
Explain why the graph of g has no points of inflexion. 1

7. The diagram below shows the graphs of $y = x$ and $y = k(x)$ where the function k is defined by $k(x) = e^{x^2} - 1$, $-1.5 \leq x \leq 1$. 4



- (a) As indicated on the diagram, the graphs intersect at the point P . Use Newton's method, with $x_0 = 0$, to find the coordinates of P correct to four decimal places. 4

- Newton's method uses the iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ to produce successive approximations to a solution of the equation $f(x) = 0$. 1

- (b) Sketch the graph of the inverse function k^{-1} . 2
(c) Find a formula for k^{-1} . 1

[Turn over

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Page three

15. A tank initially holds 20 litres of pure water. A solution of water containing 0.1 kg of salt per litre flows into the tank at a rate of 1 litre per minute. The contents of the tank are stirred continually to maintain a uniform concentration at the same time. At time t minutes, the water in the tank contains x kg of salt. 1

- (a) Write down expressions for: 2
(i) the amount of salt flowing into the tank per minute; 1
(ii) the amount of salt flowing out of the tank per minute; 1
Hence show that at any time $t > 0$, the amount of salt, x kg, in the tank can be modelled by the differential equation $\frac{dx}{dt} = \frac{2-x}{5}$. 4

- (b) Find a formula for x in terms of t . 2
(c) How much salt is present after 20 minutes? 2
(d) In the long term, what will be the amount of salt in the tank? 1

[END OF QUESTION PAPER]

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