SECTION I

Attempt ALL questions in this Section.

Marks

1. Use Gaussian elimination to solve the system of linear equations

$$x + y + z = 0$$
$$2x - y + z = -1{\cdot}1$$
$$x + 3y + 2z = 0{\cdot}9.$$ **5**

2. Let $u_1, u_2, \ldots, u_n, \ldots$ be an arithmetic sequence and $v_1, v_2, \ldots, v_n, \ldots$ be a geometric sequence. The first terms u_1 and v_1 are both equal to 45, and the third terms u_3 and v_3 are both equal to 5.

(*a*) Find u_{11}. **3**

(*b*) Given that $v_1, v_2, \ldots$ is a sequence of **positive** numbers, calculate $\sum_{n=1}^{\infty} v_n$. **3**

3. Differentiate the following functions with respect to x, simplifying your answers where possible.

(*a*) $h(x) = \sin(x^2)\cos(3x)$ **3**

(*b*) $y = \dfrac{\ln(x+3)}{(x+3)}, \quad x > -3$ **3**

4. A child's toy is pulled across the surface of a rectangular table by forces $(3\mathbf{i} + 4\mathbf{j})$ newtons and $(4\mathbf{i} - 2\mathbf{j})$ newtons, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors parallel to adjacent edges of the table. What is the magnitude of the resultant force on the toy and what angle does this resultant make with the direction of $\mathbf{i}$? **4**

5. Use the substitution $x = 4\sin t$ to evaluate the definite integral

$$\int_0^2 \frac{x+1}{\sqrt{16 - x^2}}\,dx.$$ **5**

6. A production lot of 200 cassette tapes contains 10 defective tapes.

(*a*) Write down the probability that when one tape is picked at random it is defective. **1**

(*b*) Given that a defective tape has been selected and not replaced, what is the probability that when another tape is picked at random it is also defective? **1**

(*c*) In a quality control check on the lot, two tapes are selected at random in succession and **not replaced**.

Draw a tree diagram showing all possible outcomes of the quality control check.

Hence calculate the probability that at least one of the tapes selected is defective. **4**

Marks

7. (*a*) Verify that $z = 2$ is a solution of the equation $z^3 - 8z^2 + 22z - 20 = 0$. **1**

(*b*) Express $z^3 - 8z^2 + 22z - 20$ as a product of a linear factor and a quadratic factor with real coefficients. Hence find **all** the solutions of $z^3 - 8z^2 + 22z - 20 = 0$. **4**

8. Apply the central difference method, with $h = 0{\cdot}01$, to the function $f(x) = x^2 10^x$ to obtain an approximate value for $f'(0{\cdot}5)$. Give your answer correct to three decimal places. **4**

9. Use integration by parts to obtain

$$\int_0^3 x\sqrt{x+1}\,dx.$$ **6**

10. In 1996, 30 pupils were given a class test and the marks out of 50 were recorded as follows.

Mark	24	28	30	32	35	36	38	40	42
Frequency	5	3	4	6	2	3	5	1	1

Calculate the mean mark scored in this test. **1**

The standard deviation for this set of marks was 5·14.

The following year, in an attempt to improve results, an intensive supported study programme was introduced. The same test was given to the 1997 class and this time the mean mark was 39·3 and the standard deviation was 8·2.

Comment on the success or otherwise of the supported study programme on the basis of this evidence. Justify your view. **2**

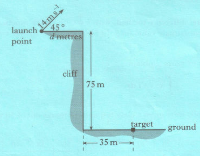
11. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$. Write down the matrix $A - \lambda I$, where $\lambda \in \mathbf{R}$ and I is the 3×3 identity matrix.

Find the values of λ for which the determinant of $A - \lambda I$ is zero. **5**

[Turn over

Marks

12. A projectile is launched with initial speed $14\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 45°. The launch point is on level ground d metres from the edge of a vertical cliff of height 75 metres, as shown in the diagram below.



The projectile strikes a target situated at a distance of 35 metres from the base of the cliff and level with the base. Determine the value of d.

(You should take the magnitude of the acceleration due to gravity to be $9{\cdot}8\,\mathrm{m\,s^{-2}}$ and also ignore air resistance.) **5**

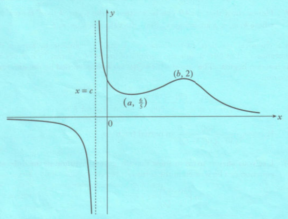
SECTION II

Attempt ALL questions in this Section.

Marks

13. The diagram below shows part of the graph of the function f, where

$$f(x) = \frac{6}{4x^3 - 12x^2 + 9x + 3}.$$



(*a*) The graph of f has a minimum turning point at $\left(a, \frac{6}{5}\right)$ and a maximum turning point at $(b, 2)$. Use calculus to obtain the values of a and b. **4**

(*b*) The line $x = c$ is a vertical asymptote to the graph of f.

(i) Write down an equation which c must satisfy. **1**

(ii) Use Newton's method, with $x_0 = -0{\cdot}2$, to find an approximation to the value of c correct to four decimal places. **4**

[Newton's method uses the iteration $x_{n+1} = x_n - \dfrac{p(x_n)}{p'(x_n)}$ to produce successive approximations to a solution of the equation $p(x) = 0$.]

[Turn over

Marks

14. Let $z = \cos\theta + i\sin\theta$.

(*a*) Use the binomial theorem to show that the real part of z^4 is

$$\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$

Obtain a similar expression for the imaginary part of z^4 in terms of θ. **5**

(*b*) Use de Moivre's theorem to write down an expression for z^4 in terms of 4θ. **1**

(*c*) Use your answers to (*a*) and (*b*) to express $\cos 4\theta$ in terms of $\cos\theta$ and $\sin\theta$. **1**

(*d*) Hence show that $\cos 4\theta$ can be written in the form $k(\cos^m\theta - \cos^n\theta) - p$ where k, m, n, p are integers. State the values of k, m, n, p. **4**

15. In a chemical reaction, two substances X and Y combine to form a third substance Z. Let $Q(t)$ denote the number of grams of Z formed t minutes after the reaction begins. The rate at which $Q(t)$ varies is governed by the differential equation

$$\frac{dQ}{dt} = \frac{(30 - Q)(15 - Q)}{900}.$$

(*a*) Express $\dfrac{900}{(30 - Q)(15 - Q)}$ in partial fractions. **2**

(*b*) Use your answer to (*a*) to show that the general solution of the differential equation can be written in the form

$$A \ln\left(\frac{30 - Q}{15 - Q}\right) = t + C,$$

where A and C are constants.

State the value of A and, given that $Q(0) = 0$, find the value of C. **4**

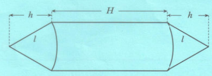
Find, correct to two decimal places,

(i) the time taken to form 5 grams of Z, **1**

(ii) the number of grams of Z formed 45 minutes after the reaction begins. **2**

Marks

16. A plastic container for holding pens is made from a cylinder with conical ends as shown in the diagram. The cylinder has radius 3 cm and length H cm. Each cone has perpendicular height h cm and slant height l cm. The total volume of the container is $900\,\mathrm{cm}^3$.



(*a*) Find an expression for H in terms of h. **3**

(*b*) Show that the surface area, $S\,\mathrm{cm}^2$, of the container is given by

$$S = 600 - 4\pi h + 6\pi\sqrt{9 + h^2}.$$ **3**

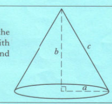
(*c*) Find the value of h for which the total surface area of the container is a minimum. Justify your answer. **5**

Note

You may use the following formulae for the volume and curved surface area of a cone with base radius a, perpendicular height b and slant height c.

Volume $= \frac{1}{3}\pi a^2 b$

Curved surface area $= \pi a c$



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