SECTION I

Attempt ALL questions in this Section.

Marks

1. Differentiate

$$g(x) = \frac{\sin x}{1+\cos x}, \qquad -\pi < x < \pi,$$

and simplify your answer. **3**

2. The noon temperatures (in °C) recorded at 22 European cities on a particular winter's day are displayed below.

2	–2	2	8	–8	–4	1	3	–2	3	2
2	1	9	–4	0	2	4	13	10	11	–11

Construct a box plot for these temperature readings. **4**

3. Use the substitution $u = t + 1$ to find

$$\int_0^8 \frac{t+2}{\sqrt{t+1}}\,dt.$$ **5**

4. A crow and a robin are flying with velocities $\mathbf{v}_c = 6\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v}_r = 4\mathbf{i} + 5\mathbf{j}$ respectively. Work out the velocity of the crow relative to the robin.

Given that distances are measured in metres and times in seconds, calculate the relative speed of the two birds. **3**

5. Use the composite trapezium rule with **four** sub-intervals to obtain an approximation to the definite integral

$$\int_0^{\frac{1}{2}} x\sin(\pi x)\,dx.$$ **4**

(Give your final answer to four decimal places.)

6. Use calculus to find all the values of x for which the function

$$f(x) = (1+x)^2 e^{-x}, \qquad x \in \mathbf{R},$$

is increasing. **5**

Marks

7. Verify that $z = 1 + i$ is a solution of the equation

$$z^3 + 16z^2 - 34z + 36 = 0.$$ **3**

Write down a second solution of the equation. **1**

Hence find constants α and β such that

$$z^3 + 16z^2 - 34z + 36 = (z^2 - \alpha z + \alpha)(z + \beta).$$ **3**

8. Show that

$$\int_0^t x\sin(\pi x)\,dx = \frac{1}{\pi^2}\sin(\pi t) - \frac{t}{\pi}\cos(\pi t).$$ **4**

Use this result to find the **exact** value of $\int_0^{\frac{1}{2}} x\sin(\pi x)\,dx$. **1**

9. Express $\frac{1}{(2x-1)(2x+1)}$ in partial fractions. **2**

Deduce that

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right).$$ **2**

Evaluate

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+1)}.$$ **1**

10. A car is travelling along a straight road at a constant velocity of $19\,\mathrm{m\,s^{-1}}$. Spotting that the road is blocked some distance ahead by a fallen tree, the driver brakes immediately and brings the car to a complete stop T seconds later. The car's acceleration t seconds after the brakes are applied is $a(t)\,\mathrm{m\,s^{-2}}$, where

$$a(t) = -\tfrac{3}{2}\sqrt{4+t}, \qquad 0 \le t \le T.$$

(*a*) Show that $T = 5$. **4**

(*b*) Calculate the distance travelled by the car during the 5 seconds that it takes to come to a stop. **3**

11. (*a*) Write down the binomial expansion of $(a+b)^4$. **1**

(*b*) On a four-day short break to a large city, I visited a tourist attraction each day chosen with equal probability from a museum, an art gallery, a shopping centre and a sports centre.

What is the probability

(i) that I never picked the museum?

(ii) that I chose the art gallery on at least three of the four days? **2, 2**

[Turn over

Marks

12. In a town with population 40 000, a 'flu virus spread rapidly last winter. The **percentage** P of the population infected t days after the initial outbreak satisfies the differential equation

$$\frac{dP}{dt} = kP, \qquad \text{where } k \text{ is a constant.}$$

(*a*) If 100 people are infected initially, find, in terms of k, the percentage infected t days later. **4**

(*b*) Given that 500 people have 'flu after 7 days, how many more are likely to have contracted the virus after 10 days? **3**

SECTION II

Attempt ALL questions in this Section.

Marks

13. Let the function f be given by

$$f(x) = \frac{2x^3 - 7x^2 + 4x + 5}{(x-2)^2}, \qquad x \neq 2.$$

(*a*) The graph of $y = f(x)$ crosses the y-axis at $(0, a)$. State the value of a. **1**

(*b*) For the graph of $f(x)$

(i) write down the equation of the vertical asymptote, **1**

(ii) show algebraically that there is a non-vertical asymptote and state its equation. **3**

(*c*) Find the coordinates and nature of the stationary point of $f(x)$. **4**

(*d*) Show that $f(x) = 0$ has a root in the interval $-2 < x < 0$. **1**

(*e*) Sketch the graph of $y = f(x)$. (You must include on your sketch the results obtained in the first four parts of this question.) **2**

14. The matrices A and B are defined by

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -1 & 9 \\ 4 & -8 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 71 & a & -6 \\ 34 & b & -3 \\ -12 & c & 1 \end{pmatrix}$$

where a, b and c are constants.

(*a*) Find the matrix $B - 3A$. **2**

(*b*) (i) Verify that $AB = I$, where I is the 3×3 identity matrix, provided that

$$\begin{aligned} a - b + 3c &= 0 \\ 2a - b + 9c &= 1 \\ 4a - 8b + c &= 0. \end{aligned}$$ **3**

(ii) Use Gaussian elimination to find the values of a, b and c for which $AB = I$. **4**

[Turn over

Marks

15. (*a*) Find the derivative of the function $g(x) = \sqrt{1-x^2}$ and hence, or otherwise, obtain the indefinite integral

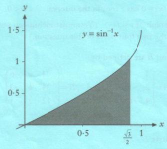
$$\int \frac{x}{\sqrt{1-x^2}}\,dx.$$ **3**

(*b*) Use integration by parts to show that

$$\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$$

where f is a differentiable function with derivative f'. **2**

(*c*) The diagram below represents the graph of $y = \sin^{-1}x$. Use the previous results to calculate the area of the shaded region which lies between $x = 0$ and $x = \frac{\sqrt{3}}{2}$. **4**



Marks

16. (*a*) A toy manufacturer is planning to launch a new product on the market. The Sales Manager believes that the recurrence relation

$$u_n = u_{n-1} + 5000 \qquad n = 2, 3, 4, \ldots$$

provides a reasonable estimate, £u_n, of the profit that will be made from sales of this product during the nth month after it is launched.

Given that $u_1 = 20\,000$, obtain simple expressions for u_n and $\sum_{k=1}^{n} u_k$ in terms of n.

Calculate the estimated profit that will be made from sales during the first year following the launch of the product. **4**

(*b*) The Managing Director objects to the Sales Manager's formula and she insists that the recurrence relation

$$v_n = 0{\cdot}9v_{n-1} + 5000 \qquad n = 2, 3, 4, \ldots$$

should be used to produce a more cautious forecast of the monthly profits.

Express v_2, v_3 and v_4 in terms of v_1. **2**

By recognising a connection with a geometric series, deduce that if $v_1 = 20\,000$ then

$$v_n = 50\,000 - 30\,000\,(0{\cdot}9)^{n-1} \qquad n = 1, 2, 3, \ldots$$

Show that $v_n < u_7$ for all n. **4**

[END OF QUESTION PAPER]