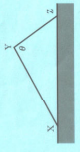
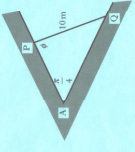


16. (a) The vertices  $X$ ,  $Y$  and  $Z$  of triangle  $XYZ$  lie on a circle of radius  $r$  cm. The side  $XY$  has length  $a$  metres and  $YZ$  length  $b$  metres. Given that  $XY$  is perpendicular to  $YZ$ , find an expression for the area of  $XYZ$  in terms of  $a$  and  $b$ . [3 marks]



(b) Two straight walls meet at  $A$  at an angle  $\frac{\pi}{4}$ . A straight piece of fencing  $PQ$  of length 10 metres is used to create an enclosure  $APQ$  as shown.



(i) Show that the area of  $APQ$  is given by  $5\sqrt{2} \sin \theta \left( \frac{1}{\sqrt{2}} - \theta \right)$ .

(ii) If the angle  $\theta$  is varied, using differentiation or otherwise, find the value of  $\theta$  which maximises the area of  $APQ$ .

[END OF QUESTION PAPER]

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Marks

1. Differentiate with respect to  $x$

(a)  $y = 2x^3 - 5x^2 + 3x - 7$ ,  $x > 0$ . [2 marks]

(b)  $y = x^2 + 1$ ,  $x > 0$ . [3 marks]

2. The point  $A$  represents  $-5 + 5i$  in an Argand diagram and  $ABCD$  is a square with centre  $-2 + 2i$ . Find the complex numbers represented by the points  $B$ ,  $C$  and  $D$ , giving your answers in the form  $x + iy$ . [4 marks]

3. 

|   |   |   |   |   |
|---|---|---|---|---|
| 5 | 2 | 1 | 3 | 4 |
| 6 | 4 | 1 | 0 | 5 |
| 8 | 7 | 2 | 3 | 1 |
| 9 | 3 | 4 | 4 | 3 |

The stem-and-leaf diagram represents sales of compact discs (CDs) over 19 days.

(a) Calculate the mean number of CDs sold. [2 marks]

(b) Work out the median number of CDs sold. [1 mark]

(c) On how many days were less than 62 CDs sold? [1 mark]

4. A trolley is being pulled across a horizontal surface by forces  $F_1 = 5i + 2j$  newtons and  $F_2 = 4i - 3j$  newtons where  $i$  and  $j$  are perpendicular unit vectors.

(a) What is the magnitude of the resultant force which acts on the trolley? [2 marks]

(b) What angle does the resultant make with the direction of  $F_1$ ? [2 marks]

5. Use Gaussian elimination to solve the system of equations.

$$\begin{cases} x - y + z = 2 \\ 2y - z = 6 \\ 2x + 3y + z = 1 \end{cases}$$

(6) Use Newton's method with  $x_0 = 1$  to find an approximation to the root of  $e^x + x - 2 = 0$  correct to four decimal places. [4 marks]

(Newton's method uses the iteration  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$  to solve the equation  $f(x) = 0$ .)

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7. I have just received a letter from Spanish. Condensed Double-Clicking says that I have won a prize in their contest. They tell me that I have won an amount for the rest of my life and have arranged for the prize to be paid as follows.

At the end of the first year, I will receive £20 000, at the end of the second year, £20 000, at the end of the third year, £14 000 and so on, with the amount increasing by £2000 each year.

What will my total prize money amount to at the end of the  $n$ 'th year? [3 marks]

How many years will I have to live before the accumulated prize money exceeds £200 000? [2 marks]

8. (a) By using the substitution  $u = 2 \sin x$ , or otherwise, evaluate the definite integral  $\int_0^{\pi/6} \frac{\cos x}{1 + \sin^2 x} dx$ . [4 marks]

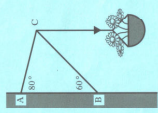
(b) Use integration by parts to find  $\int_0^1 x^2 \ln x dx$ . [3 marks]

9. As shown in the diagram, a framework of light rods with frictionless joints is attached to a wall and supports a hanging flower basket of mass  $M$  kilograms.

(a) By resolving horizontally and vertically, or otherwise, calculate in terms of  $M$  the tension in the rod  $BC$ . [5 marks]

(b) The rod  $BC$  will buckle if it is subjected to a compression of 100 newtons or more. Work out the least mass of hanging basket which will cause this to happen. [2 marks]

(c) It would be interesting to know the magnitude of the accelerations due to gravity to be  $9.81 \text{ m s}^{-2}$ .



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[Turn over]

10. Solve the equation  $x^2 + \sqrt{2}x + 4 = 0$  for the complex number  $z$ . Give the modulus and argument of each of the roots and illustrate them on an Argand diagram. [5 marks]

(b) Each day of my working week, from Monday to Friday, I have a coin of a certain value which I toss. If it shows a number from 1 to 5, I get a supermarket trolley. [1 mark]

(i) For each  $k = 0, 1, 2, 3, 4, 5$  write down the probability that I choose the supermarket trolley for  $k$  days out of the 5. [3 marks]

(ii) What is the probability that over two working weeks my choice is always the same? [1 mark]

12. Let  $f$  be the function given by  $f(x) = 4x^3 - 5x - 1$ . Find algebraically the values of  $t$  for which the slope of this function is between  $-3$  and  $3$ . [5 marks]

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Marks

13. When a cube is opened, the rate at which the water drains from a pool is proportional to the square root of the depth of the water.

This can be represented by the differential equation  $\frac{dh}{dt} = -\sqrt{h}$ ,  $h > 0$ , where  $h$  is the depth (in metres) of the water and  $t$  is the time (in minutes) since the pool was opened.

(a) Express  $h$  as a function of  $t$ . [4 marks]

(b) Find the solution of the equation, given that the pool was initially 4 m deep. [2 marks]

(c) The exact time the pool had to be drained, the water was initially 9 m deep. How long did it take to drain the pool on this occasion? [3 marks]

14. (a) (i) Show that  $\int_0^1 \sqrt{1-x^2} dx = \frac{\pi}{4} - \frac{1}{2}$ . [2 marks]

(ii) Hence find  $\int_0^1 \sqrt{1+x} dx$  correct to 3 decimal places. [2 marks]

(b) Use the composite trapezium rule with  $h = 0.25$  to find  $\int_0^1 \sqrt{x+1} dx$  correct to 3 decimal places. [4 marks]

(c) Which method, the one in part (a) or the one in part (b), is preferable and why? [1 mark]

15. (a) Find a real root of the cubic polynomial  $6x^3 + x^2 - x - 2$  and hence decrease it as a product of a linear term  $l(x)$  and a quadratic. [3 marks]

(b) Show that  $l(x)$  cannot be written as a product of three real linear factors. [1 mark]

(c) Use your factorisation to find values of  $A$ ,  $B$  and  $C$  such that  $\frac{5x+4}{x^3-x^2+x-2} = \frac{A}{l(x)} + \frac{Bx+C}{q(x)}$ .

Hence obtain the indefinite integral  $\int \frac{5x+4}{x^3-x^2+x-2} dx$ .

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[Turn over]