SECTION I

Attempt ALL questions in this Section.

Marks

1. Differentiate the functions

   (*a*) $\dfrac{\sin x}{x}$, $x \neq 0$, — 2

   (*b*) $\tan^{-1}(2\cos x)$. — 3

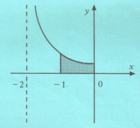
2. In an archery competition, the probability that a particular competitor hits the target with a shot is $\frac{3}{4}$. In the competition, she is allowed three shots. Work out the probabilities that she:

   (*a*) hits the target exactly twice; — 3

   (*b*) hits the target at least once. — 2

3. (*a*) Express $\dfrac{3}{(x+2)(x+5)}$ in partial fractions. — 3

   (*b*) The diagram below represents the graph of $y = \dfrac{3}{(x+2)(x+5)}$ for $-2 < x \leqslant 0$.



   Evaluate the shaded area. — 4

4. Newton's method for improving an approximation to a root of the equation $f(x) = 0$ uses the recurrence relation

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

   Use Newton's method to obtain an approximation, correct to three significant figures, to the root of the equation

   $$\cos x - 2x = 0.$$ — 4

Marks

5. Find the value(s) of λ for which the determinant of the matrix

   $$A = \begin{pmatrix} 2 & 3 & 4 \\ -1 & \lambda & 0 \\ 1 & 5 & -1 \end{pmatrix} \text{ is zero.}$$ — 3

6. A ship, A, is moving with speed 7 kilometres per hour due south and another ship, B, is moving with speed 15 kilometres per hour due west. Find the velocity of A relative to B. — 4

7. (*a*) By equating the coefficients of the powers of x, or otherwise, find real numbers p and q such that

   $$x^4 - x^2 - 2x - 1 = (x^2 + px - 1)(x^2 + qx + 1).$$ — 3

   (*b*) Hence find all four solutions of the equation

   $$x^4 - x^2 - 2x - 1 = 0$$

   and show them on an Argand diagram. — 4

8. Apply the central difference method, with $h = 0{\cdot}001$, to the function $f(x) = x\ln x$ to obtain an approximation to $f'(3)$. Give your answer correct to four decimal places. — 3

9. The performance of a prototype of a surface-to-air missile was measured on a horizontal test bed at the firing range and it was found that, until its fuel was exhausted, its acceleration (measured in m s^{-2}) t seconds after firing was given by

   $$a = 8 + 10t - \tfrac{3}{4}t^2.$$

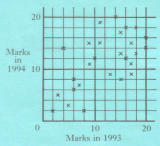
   (*a*) Obtain a formula for its speed, t seconds after firing. — 3

   (*b*) The missile contained enough fuel to fire for 10 seconds. What horizontal distance would it have covered on the firing range when its fuel was exhausted? — 3

[Turn over

Marks

10. A group of 27 pupils took two Mathematics tests, one in 1993 and the other in 1994. Their marks out of 20 are shown on the "scatter diagram" below. So, for example, the cross at (15, 17) represents a pupil who scored 15 in 1993 and 17 in 1994.



   Use the information contained in the diagram to sketch a box plot of the results of the 1993 examination. — 4

   (You do not need to draw an accurate scale diagram, but you should mark on your sketch the values of the median and of the upper and lower quartiles and also the ranges.)

11. Use the substitution $x = 1 - \sin\theta$ to evaluate the definite integral

   $$\int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{2x - x^2}}.$$ — 5

12. Obtain the derivative of the function

   $$f(x) = x\ln x \qquad (x > 0).$$ — 2

   Hence, or otherwise, find the indefinite integral $\int \ln x\,dx$. — 2

   Work out the **exact** value of

   $$\int_2^4 \ln x\,dx,$$

   expressing your answer in terms of ln 2. — 3

SECTION II

Attempt ALL questions in this Section.

Marks

13. Consider the function $f(x) = \dfrac{x^2 + 3x + 6}{x + 2}$, $x \in \mathbf{R}$, $x \neq -2$.

   (*a*) Show that $y = f(x)$ has a non-vertical asymptote and obtain its equation. — 3

   (*b*) Find the coordinates of the turning points of $f(x)$ and justify their nature. — 4

   (*c*) Sketch the graph of $y = f(x)$, indicating all asymptotes and stationary values. — 2

   (*d*) For what values of k does the equation $f(x) = k$ have no real solutions? — 2

14. For any angle x, define A_x and B_x to be matrices

   $$A_x = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}, \quad B_x = \begin{pmatrix} \cos x & \sin x \\ \sin x & -\cos x \end{pmatrix}.$$

   (*a*) Show that $(B_x)^2 = I$, where I is the 2×2 identity matrix. — 2

   (*b*) Show that, for any angles r, s,

   $$A_r A_s = A_{r+s}.$$ — 2

   (*c*) Show also that, for any angles r, s, there is an angle t such that

   $$A_r B_s = B_t,$$ — 3

   and that for this angle t it is also true that

   $$B_s B_r = A_t.$$ — 3

   [You may use the identities: $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
   $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$.]

15. (*a*) Use integration by parts to evaluate

   $$\int_0^{\pi} x\sin x\,dx.$$ — 3

   (*b*) Let $Q_n = \displaystyle\int_0^{\pi} x^n \sin x\,dx$, where n is a positive integer.

   Show that, for $n \geqslant 3$,

   $$Q_n = \pi^n - n(n-1)Q_{n-2}.$$ — 5

   (*c*) Use this result to find the value of $\displaystyle\int_0^{\pi} x^5 \sin x\,dx$. — 2

[Turn over

Marks

16. It is given that

   $$1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x}, \quad x \neq 1.$$

   (*a*) By differentiating this identity with respect to x, or otherwise, obtain a formula for the sum of the series

   $$1 + 2x + 3x^2 + \ldots + nx^{n-1}.$$ — 3

   (*b*) Let $S(n) = 2 + 3x + 4x^2 + \ldots + (n+2)x^n$.

   By making use of the earlier series, or otherwise, show that $S(20)$ can be expressed in the form

   $$S(20) = \frac{ax^{22} + bx^{21} + cx + d}{(x-1)^2}, \quad \text{where } a, b, c \text{ and } d \text{ are constants.}$$

   State the values of a, b, c and d. — 6

[END OF QUESTION PAPER]