SECTION I

Attempt ALL the questions in this Section.

Marks

1. After time t, the position vector of a particle P is $(4t+21)\mathbf{i}+(8+11t-4t^2)\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions of rectangular axes Ox and Oy respectively. A second particle Q has acceleration $4\mathbf{i}$. At time $t=0$, Q has position vector $6\mathbf{i}+3\mathbf{j}$ and velocity vector $5\mathbf{i}+3\mathbf{j}$. Show that the particles will collide. **4**

2. A planet, radius 25 000 kilometres, rotates about its axis once every 14 hours. The magnitude of the gravitational acceleration on the surface of the planet is $10\cdot8\,\mathrm{m\,s^{-2}}$. A satellite moves in such a way that it remains at the same height above a fixed point on the equator of the planet. Calculate this height, assuming that Newton's inverse square law of gravitation applies. **6**

3. At time t, particles P and Q have position vectors given by $\mathbf{r}_\mathrm{P}=a\cos(\omega t)\mathbf{i}+b\sin(\omega t)\mathbf{j}$ and $\mathbf{r}_\mathrm{Q}=-b\sin(\omega t)\mathbf{i}+a\cos(\omega t)\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions of rectangular axes Ox and Oy respectively and a, b and ω are positive constants. Show that the acceleration of P relative to Q is proportional to the position vector of P relative to Q. **4**

4. A gun of mass M fires a shell of mass m horizontally. The gun recoils horizontally and experiences a constant resistance, R, which brings it to rest in a distance d. Given that the shell has an initial speed v, show that the magnitude of R is given by

$$R=\frac{(mv)^2}{2Md}.$$ **4**

5. A cyclist ascends a track of constant gradient in 30 minutes. The track is 8 kilometres long and rises a vertical distance of 300 metres. The cyclist experiences a resistance of magnitude $(0\cdot2v^2+2\cdot5)$ newtons, where v is the speed of the cyclist in metres per second. The total mass of the cyclist and cycle is 70 kilograms. Assuming the speed of ascent to be constant, calculate the power at which the cyclist is working. **5**

6. A lift of mass M is at rest supported by a cable. The tension in the cable remains constant at all times. A tile, mass m, becomes detached from the ceiling of the lift, causing the lift to move vertically upwards as the tile falls.

   Find the acceleration of the tile relative to the lift. **5**

   The height of the ceiling in the lift is h. Show that the time t taken for the tile to hit the floor is given by

$$t=\sqrt{\frac{2(M-m)h}{Mg}},$$

   where g is the magnitude of the acceleration due to gravity. **2**

SECTION II

Attempt THREE questions from this Section

Each question is worth 15 marks.

Marks

7. A particle is projected from the origin O, under constant gravity of magnitude g, with a velocity whose horizontal component has magnitude U, and whose upward vertical component has magnitude V.

(*a*) Show that, relative to horizontal and vertical axes Ox and Oy respectively, the path of the projectile is given by the equation

$$2U^2y = 2UVx - gx^2.$$ **4**

(*b*) Given that the particle passes through the points $(4, 7{\cdot}2)$ and $(10, 15)$, find the angle of projection to the horizontal. **6**

(*c*) Find the coordinates of the highest point reached by the particle on this path. **5**

8. A missile, of mass m, is projected vertically upwards under constant gravity. During its flight, the missile experiences a resistive force of magnitude kv^2 per unit mass, where v is the speed of the missile and k is a positive constant.

(*a*) Given that the initial speed of the missile is U, find an expression for the maximum height reached by the missile in terms of k, U and g, where g is the magnitude of the acceleration due to gravity. **8**

(*b*) Show that, when the missile has fallen back to its starting point, the speed of the missile is given by

$$U\sqrt{\frac{g}{g + kU^2}}\,.$$ **7**

9. On a test bed, a vertical car spring stands on a horizontal plate which is level with a point O. A mass of 100 kilograms, attached to the top of the spring as a load, compresses the spring by 5 centimetres. Find the stiffness constant of the spring. **2**

To simulate travelling along a road with the profile of a sine curve, the horizontal plate is made to oscillate vertically, starting from a position level with O. The oscillations of the plate have amplitude 6 centimetres and period $\frac{2\pi}{7}$ seconds. Show that the height, $r(t)$ metres, of the oscillating plate above level O after time t seconds, is given by $r(t) = 0{\cdot}06\ \sin(7t)$. **1**

Show that the height, $y(t)$ metres, of the load above its equilibrium position at time t seconds, satisfies the differential equation

$$\frac{d^2y}{dt^2} + 196y = 11{\cdot}76\ \sin(7t).$$ **4**

Find the solution of this equation for which $y = 0$ and $\frac{dy}{dt} = 0$ when $t = 0$. **8**

[Turn over

Marks

10. A bend on a racetrack is circular and is banked at an angle α to the horizontal such that a car can take the bend at speed V with no sideways friction between the wheels and the track. Express V in terms of α, r and g, where r is the radius of the bend and g is the magnitude of the acceleration due to gravity. **3**

A car taking the bend at speed v is on the point of skidding outwards. Show that

$$v^2 = \frac{gr(\tan\alpha + \mu)}{1 - \mu\tan\alpha},$$

where μ is the coefficient of friction between the car tyres and the surface of the track. **4**

In dry conditions, $v = 2V$ and $\mu = \frac{3}{4}$. Use the previous results to find the value of $\tan\alpha$. **3**

In wet conditions, the coefficient of friction is reduced to a new value μ', such that a car at rest on the wet track is on the point of slipping **down** the banking. Find the value of μ'. **2**

Show that the maximum speed for taking the bend without skidding in wet conditions is approximately 82% of the maximum speed in dry conditions. **3**

Marks

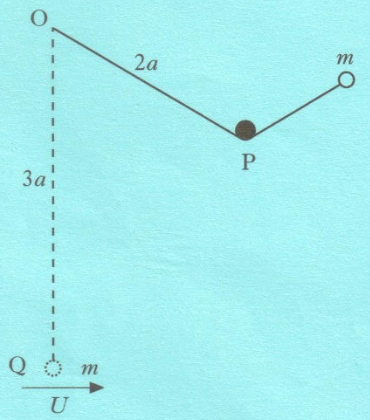
11. A particle of mass m is attached at one end of a light, inextensible string of length $3a$. The other end of the string is fixed at the origin, O. The particle is hanging in equilibrium at Q, vertically below O, and is propelled horizontally from this position with initial speed U.

(*a*) The speed of the particle is V when the taut string makes an angle θ with the downward vertical, OQ. Use the principle of Conservation of Energy to find an expression for V^2 in terms of U, θ, a and g, where g is the magnitude of the acceleration due to gravity. **3**

(*b*) Show that the particle will describe complete circles about O, with the string taut, provided

$$U^2 \geq 15ga.$$ **6**

A smooth peg, P, is now positioned at a distance $2a$ from O such that angle $QOP = \frac{\pi}{3}$ radians and the mass is propelled from Q with speed U as before.



(*c*) Find the speed of the particle when the string first comes into contact with the peg. **1**

(*d*) Find a condition similar to that in (*b*) for the particle to complete a loop round the peg. **5**

[*END OF QUESTION PAPER*]