SECTION I

Attempt ALL the questions in this Section.

Marks

1. A body of mass 4 kg starts from rest at the origin and moves in a straight line until it is again at rest. Its velocity, in m s^{-1}, after t seconds is given by $2t(3 - \sqrt{t})\mathbf{i}$, where $\mathbf{i}$ is the unit vector in the direction of motion. Find the maximum speed of the body and the work done in reaching this maximum speed. **5**

2. A lift of mass 500 kg moves in a vertical shaft, subject to constant frictional resistance of 100 newtons. An engine, working at 10 kW, pulls the lift upwards at uniform speed. Calculate this speed. **3**

Given that the engine continues to work at the same rate and the resistance remains as before, find the acceleration of the lift when being pulled upwards with speed 1·5 m s^{-1}. **2**

3. A particle is moving in a horizontal circle, centre the origin, with constant angular speed. Its position vector after time t is given by

$$\mathbf{r} = 5\cos(4t)\mathbf{i} + 5\sin(4t)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions of two fixed, perpendicular radii. Find the velocity vector $\mathbf{v}$ of the particle after time t, and show that this velocity vector is perpendicular to the position vector throughout the motion. **3**

4. A coastguard observes that a liner is cruising due West at 20 knots and a cargo ship is sailing at 15 knots on a bearing of 330°. At mid-day the liner is 8 nautical miles due North of the cargo ship. Find the velocity of the cargo ship relative to the liner. **3**

Given that they both continue to sail with the stated speeds and directions, calculate the time when they are nearest to each other. **4**

5. A pile of mass 1000 kg is driven into the ground by a piledriver of mass 5000 kg, which is dropped vertically on to the pile from a height of 4 metres. The piledriver falls freely on to the pile, remaining in contact with the pile after the impact, and drives the pile 25 centimetres into the ground.

Calculate

(i) the speed with which the pile starts to penetrate the ground; **2**

(ii) the resistance of the ground to penetration, assuming this resistance to be constant. **3**

6. A body of constant mass m is projected vertically upwards from the Earth's surface with initial speed U. The only force acting on the body is that due to the Inverse Square Law of Gravitation. Given that the maximum height reached by the body is R, where R is the radius of the Earth, find U in terms of R and g, the magnitude of the acceleration due to gravity at the surface of the Earth. **5**

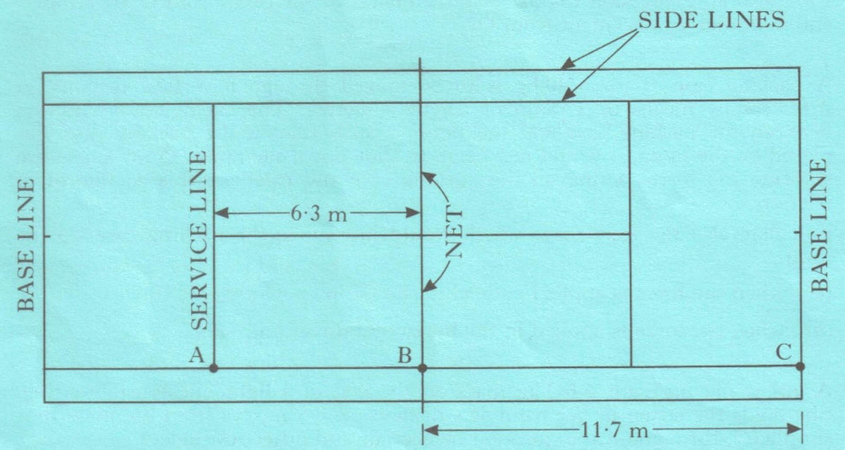
SECTION II

Attempt THREE questions from this Section.

Each question is worth 15 marks.

Marks

7. A tennis player hits an overhead volley when standing at A, the point of intersection of the service line and the inner side line. The distance from A to the net is 6·3 metres. The ball is hit, straight down the side line AC, from a height of 2·8 metres vertically above A. The ball clears the net, which is 1 metre high at B. The base line is 11·7 metres from the net.



(*a*) The ball is projected horizontally. Find the range of possible values of the speed of projection for the ball to land within the base line. **8**

(*b*) Now suppose that the ball is projected with speed $30\ \text{m s}^{-1}$, at an angle α **below** the horizontal. Given that the ball hits the ground **on** the base line at C, find the angle α. **7**

$$\left[\text{Note that } \frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha \text{ when } \cos\alpha \neq 0.\right]$$

[Turn over

Marks

8. A body moves in a horizontal, straight line in the direction Ox, in a medium which offers a resistance whose magnitude per unit mass is twice the speed of the body. It is also acted on by a constant braking force whose magnitude is four times the mass of the body. The initial velocity of the body is in the direction Ox. After 1 second the body is at O and its velocity is 2 m s^{-1} in the direction Ox.

Write down an equation of motion for the body and find an expression for the speed v in terms of the time t. **8**

Hence calculate the initial speed of the body. **1**

Find also an expression for the displacement x of the body from O after time t and its initial displacement from O. **6**

9. A packing case of mass 40 kg is to be moved through a vertical distance of 2 metres by means of a ramp of length 6 metres. The coefficient of friction between the packing case and the ramp is such that, if the packing case was placed on the ramp, it would just begin to slide down the ramp. Draw a diagram showing the forces acting on the packing case and calculate this coefficient of friction. **3**

Calculate also the least force which would just move the packing case **up** the ramp,

(*a*) when the force is applied parallel to the ramp; **5**

(*b*) when the force is applied in the horizontal direction. **7**

10. A girl is playing with a 0·2 kg mass on the end of a light, inextensible string. She holds the string in her hand at a distance of 0·5 metres from the mass. For simplicity, throughout the question neglect air and other resistances.

First she holds her arm straight out and swings the mass so as to form a conical pendulum. The mass describes horizontal circles below her hand, with uniform speed, such that the string makes an angle of 30° with the vertical. Find the tension in the string and the time for **one** complete revolution. **6**

Next she holds her hand out with the mass hanging vertically. With her other hand she pulls the mass aside, keeping the string taut, until the string makes an angle of 60° with the downward vertical. She releases the mass from rest in this position, so as to form a simple pendulum. When the mass is at its lowest position, find its speed and the tension in the string. **6**

Finally she swings the mass in a vertical circle. Given that the breaking strain of the string is 10 newtons, and that the string breaks when the mass is at its lowest point, calculate the speed of the mass when the string breaks. **3**

Marks

11. An experiment is being carried out to test the effectiveness of shock absorbers. A body of unit mass is supported in equilibrium by a spring of stiffness constant 2. The body is then forced vertically up and down by an external periodic force of 10 cos t, where t is the time from the instant this force is first applied. A shock absorber provides a damping force of magnitude three times the speed of the vertical motion of the body. Show that the vertical displacement $x(t)$ of the body from its equilibrium position can be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 10\cos t$$

and find the general solution of this differential equation. **8**

Find also the solution of the differential equation satisfying the given initial conditions. **4**

Describe the long term behaviour of your solution and calculate the amplitude of the forced oscillations. **3**

[*END OF QUESTION PAPER*]