SECTION I

Attempt ALL the questions in this Section.

Marks

1. A girl pulls a sledge of mass 12 kg across horizontal ground which is covered with snow. The coefficient of friction between the sledge and the snow is $\frac{1}{6}$. The rope with which she is pulling the sledge makes an angle of 30° with the horizontal and the sledge moves at a steady speed. Indicate in a diagram the forces acting on the sledge and find the tension in the rope. **5**

2. A car travelling at 12 m s^{-1} starts to accelerate 40 metres before leaving a built-up area. The acceleration of the car t seconds later is given by $\frac{1}{3}(13 - 2t)$ m s^{-2}. How long will it be before the speed of the car first reaches 26 m s^{-1} and how far outside the built-up area will the car then be? **5**

3. A piston oscillates in a cylinder of length 16 cm, using the full length of the cylinder for its motion. Given that the oscillations can be modelled by simple harmonic motion, find the greatest possible number of oscillations per second if the speed of the piston must not exceed 10 m s^{-1}. **3**

4. Consider a coordinate system based on rectangular axes Ox, with unit vector $\mathbf{i}$, and Oy, with unit vector $\mathbf{j}$. In this question, distances are measured in metres and forces in newtons.

(*a*) Find the work done when the point of application of the force $\mathbf{F} = 3\mathbf{i} + 2\mathbf{j}$ moves in a straight line from the point with position vector $2\mathbf{i} + \mathbf{j}$ to the point with position vector $6\mathbf{i} + 5\mathbf{j}$. **2**

(*b*) A particle is constrained to move along the x-axis under the action of a force $(2x - 3)\mathbf{i} + 2x\mathbf{j}$. Find the work done when the point of application of the force moves from $x = -2$ to $x = 6$. **3**

5. Two helicopter pilots are practising maintaining a straight course at a constant speed at a height of 100 metres above the surface of the sea. One pilot, flying due West at 100 kilometres per hour, flies over an anchored aircraft carrier. Fifteen minutes after this instant, the second pilot is 30 kilometres due South of the aircraft carrier and is supposed to be flying at 160 kilometres per hour in a direction 65° West of North. Unfortunately, although his speed is correct, he is a bit off course, with the potential tragic consequence that the two helicopters will collide unless evasive action is taken by one of the pilots.

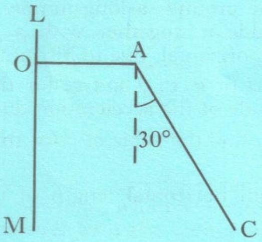
Calculate,

(*a*) the time from the instant the first pilot flies over the aircraft carrier until the collision would happen if no evasive action were taken; **5**

(*b*) the angle by which the second pilot is off course. **1**

Marks

6. At a fairground the "chair o' planes" are small chairs attached by chains 2·5 metres long to the ends of horizontal arms 1·5 metres long, radiating from a central pillar about which the arms rotate. To simplify the model, ignore the weight of the chains and consider a stage in the motion when the horizontal arms are rotating uniformly with the chains making an angle of 30° to the vertical, in the vertical plane containing the horizontal arm, as shown in the diagram.



LM represents the central pillar, OA a horizontal arm and AC the chain attaching a chair to this arm. Assume that the chain AC is in the vertical plane containing the arm OA.

For the particular stage of the motion described above in this simplified model, calculate the speed of the chairs and the time for one complete revolution. **6**

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SECTION II

Attempt THREE questions in this Section.

Each question is worth 15 marks.

Marks

7. At the instant of leaving the ground, a long-jumper has a horizontal velocity, due to his run, of magnitude u, together with a velocity, of magnitude v, inclined at an angle θ to the horizontal, due to his jump.

Find an expression, in terms of u, v, θ and g, for the horizontal length of his jump, where g is the magnitude of the acceleration due to gravity. **7**

Given that $u = v = 5{\cdot}5\ \text{m s}^{-1}$, find the value of θ for this horizontal distance to be a maximum. **4**

Calculate both the maximum horizontal length of the jump and the greatest height reached in the jump. **4**

8. A skydiver, jumping from a hovering helicopter, falls from rest under gravity and against an air resistance of kv^2 per unit mass, where v is the speed of the skydiver and k is a positive constant. Write down a differential equation for the motion and find an expression for the terminal speed U of the skydiver. **3**

Hence show that the time taken to acquire speed v can be expressed as

$$t = \frac{U}{2g}\ln\left(\frac{U+v}{U-v}\right),$$

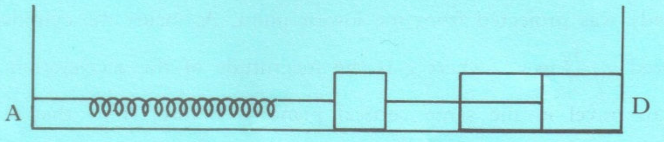
where g is the magnitude of the acceleration due to gravity. **6**

Given that the terminal speed is $54\ \text{m s}^{-1}$, show that the skydiver acquires a speed of $\frac{4}{5}U$ in just over 6 seconds. **2**

Find also the distance fallen by the skydiver in reaching this speed. **4**

Marks

9. A block of mass 0·15 kg is able to slide on a smooth, horizontal table. It is attached by a spring to a fixed point A and is connected to a damping device D by means of a rigid piston moving in a fixed pot, which contains a heavy oil.



The stiffness of the spring is 0·2 N m^{-1} and the magnitude of the resistance due to the damping device is $0{\cdot}4v$ newtons, where v metres per second is the speed of the block. The motion is started with the spring extended by 5 centimetres, by projecting the block horizontally with speed 14 centimetres per second towards A.

Let x metres denote the displacement of the block in the horizontal direction towards D, measured from the position of zero extension of the spring. Given that it is acceptable to use Hooke's Law for the tension exerted by the spring, show that a differential equation for the motion is

$$3\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 4x = 0.$$ **2**

Find the particular solution of this equation satisfying the given initial conditions at $t = 0$. **6**

Show that the block comes to rest momentarily after $\frac{3}{4}\ln 8$ seconds and describe the subsequent motion. **4**

Suppose instead that the damping device had a resistance of $0{\cdot}1v$ newtons. Write down the differential equation for this situation and describe briefly how the motion differs from that in the previous case. **3**

10. Two skaters approach each other at right angles with speeds 2·5 m s^{-1} and 3 m s^{-1} respectively. After colliding, they hold on to each other, and their momentum carries them in a direction making an angle of 60° with the original course of the slower skater.

Calculate the ratio of the masses of the two skaters. **5**

Find the speed at which the skaters move off together after the collision. **2**

By calculating the ratio of the kinetic energy after the collision to the total kinetic energy before the collision, show that a little less than half the kinetic energy is lost in the collision. **4**

Calculate the magnitude and direction of the impulse experienced by the slower skater, whose mass is 50 kg. **4**

[Turn over

Marks

11. A hollow circular cylinder of radius 0·4 metres is fixed with its axis horizontal. A body of mass 0·5 kg moves on the inside smooth surface of the cylinder and in a vertical plane perpendicular to the cylinder's axis. Find the least speed which the body must have at the lowest point of its path if it travels in complete circles. **6**

If the body was projected from the lowest point, A, inside the cylinder, with initial speed $\sqrt{\frac{3g}{2}}$ m s^{-1}, where g is the magnitude of the acceleration due to gravity, to travel in the same vertical plane as before, show that it would leave the cylinder at a point such that the radius of the circle to that point makes an angle of $\cos^{-1}\frac{7}{12}$ with the upward vertical. **5**

Find also the greatest vertical height above the lowest point A that the body would reach in its subsequent motion. **4**

[*END OF QUESTION PAPER*]