SECTION I

Candidates should attempt ALL the questions in this Section.

Marks

1. A curler wishes to hit an opponent's stone. Find the minimum speed with which the curler's stone must be projected horizontally towards the opponent's stone for the curler to succeed, given that the coefficient of friction between a curling stone and the level surface of the ice is 0·04 and that the distance from the point of release of the curler's stone to the opponent's stone is 28 metres. **3**

2. At 10 a.m., two ships P and Q are observed at positions $6\mathbf{i} + 15\mathbf{j}$ and $3\mathbf{i} + 6\mathbf{j}$ respectively, referred to rectangular axes Ox with unit vector $\mathbf{i}$ and Oy with unit vector $\mathbf{j}$. Relative to the origin O, P is travelling with velocity $11\mathbf{i} + 6\mathbf{j}$ and Q with velocity $15\mathbf{i} + 18\mathbf{j}$.

Show that the ships are on a collision course and state the time at which they will collide if neither alters course. **5**

Find also the angle between their velocity vectors while on this collision course. **2**

(Distances are measured in kilometres and times in hours.)

3. A bullet of mass 30 grams, travelling horizontally at $100\,\text{m}\,\text{s}^{-1}$, hits a stationary block of wood of mass 3 kg which is free to move on a smooth, horizontal plane. Stating any assumptions being made, find the final speed of the block in each of the cases when

(*a*) the bullet becomes embedded in the block; **3**

(*b*) the bullet passes right through the block and emerges with speed $50\,\text{m}\,\text{s}^{-1}$. **2**

Calculate the loss in kinetic energy in the latter case. **2**

4. A satellite moves with constant speed $3360\,\text{m}\,\text{s}^{-1}$ in a circular orbit round Mars. If it takes 118 minutes to complete one orbit, calculate its height above the surface of Mars, whose radius is 3397 km. **3**

Assume that the magnitude of the gravitational force varies inversely as the square of the distance from the centre of Mars and that the mass of the satellite is 100 kg. Calculate the magnitude of the force due to gravity experienced by the satellite if the acceleration due to gravity at the surface of Mars is $3{\cdot}7\,\text{m}\,\text{s}^{-2}$. **3**

5. A bead is threaded on a circular hoop and is free to move around the hoop, the contact between the bead and the hoop being smooth. The hoop, having radius a, is fixed in a vertical plane. Its highest point is A and its lowest is B.

In an experiment, the bead is projected from B with speed u. It makes complete revolutions and the speed of the bead at A is $\frac{2}{3}u$. Show that $u = 6\sqrt{ga/5}$, where g is the acceleration due to gravity. **3**

In a second experiment, the bead is projected from B with speed $\sqrt{5ga}$. Find the angle to the downward vertical through which the radius from the centre of the hoop to the bead will turn until the bead acquires the same speed as it attained at A in the first experiment. **4**

SECTION II

Attempt THREE questions from this Section.

Each question is worth 15 marks.

Marks

6. While playing cricket, a fielder lunged to retrieve the ball and threw it from ground level in such a way that it fell at the feet of the wicket-keeper 50 metres away. Find a connection between the horizontal and vertical components of the initial velocity of the ball. **4**

Had the wicket-keeper advanced 5 metres towards the fielder, the wicket-keeper would have caught the ball at a height of 1·6 metres above the ground. However he chose to remain at the wicket. Find the time of flight of the ball through the air. **8**

Find also the angle to the horizontal at which the ball was thrown. **3**

7. A train of mass 400 tonnes begins to ascend a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{350}$. Its speed at the start of the ascent is $3\,\mathrm{m\,s^{-1}}$ while its locomotive is working at 480 kW against gravity and additional constant resistances of 72 N per tonne.

Assuming that the train's locomotive continues to work at the same constant rate, show that an equation of motion is

$$\frac{dv}{dt} = \frac{12 - v}{10v},$$

where v, measured in $\mathrm{m\,s^{-1}}$, is the speed at time t seconds. **6**

Hence, or otherwise, find the maximum speed that the train can reach. **1**

Find also the time taken to reach $\frac{3}{4}$ of this maximum speed and the distance travelled up the incline in doing so. **8**

8. A set of scales consists of a light helical spring of natural length 15 cm fixed upright on a table, with a small platform of mass 12 grams fixed on top of the spring.

When this platform is depressed and released, it vibrates vertically with simple harmonic motion, completing 4 oscillations per second. Find the modulus of elasticity of the spring. **7**

A body of mass 3 grams is placed on the platform, which results in the total compression of the spring being d cm. Calculate d and find the reaction between the body and the platform immediately on release. **8**

9. A car travelling at $21\,\mathrm{m\,s^{-1}}$ has no tendency to slip while going round a circular bend of radius 180 m on a road banked at an angle θ to the horizontal. Calculate θ. **5**

The speed of the car is increased until the car is on the point of slipping **up** the slope. Given that the coefficient of friction between the car and the road is 0·2, calculate this speed. **7**

By modifying your working, find the corresponding speed for the case when the car is on the point of slipping **down** the slope. **3**

[Turn over

Marks

10. A machine part of mass 0·5 kg is vibrating at the end of a spring by which it is suspended from a fixed point. If there were no resistances to the motion, the machine part would perform simple harmonic oscillations of period $\frac{\pi}{5}$ seconds. However, the motion is resisted by a damping force of $8v$ newtons when the speed is v metres per second. Write down a differential equation for this motion and find the period of the damped oscillations. **7**

If the motion of the machine part starts from rest at a distance of 2·5 cm below its equilibrium position, find expressions for the displacement and velocity (relative to the equilibrium position) at time t seconds from the start. **6**

An adaptation to the machine results in the machine part experiencing an additional force, which acts in the vertical direction and has the periodic form $5\sin(12t)$ newtons. Write down a differential equation to model this modified situation and indicate briefly how you would attempt to solve this equation. **2**

[*END OF QUESTION PAPER*]