Marks

15. (*a*) State the binomial expansion of $(a + b)^5$. 1

(*b*) de Moivre's Theorem states that

$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$$

for any integer n.

(i) By using de Moivre's Theorem and by equating imaginary parts, show that $\sin 5\theta$ can be expressed in the form

$$k\cos^4\theta\sin\theta + l\cos^2\theta\sin^3\theta + m\sin^5\theta$$

for some real values of k, l and m.

Write down the values of k, l and m. 5

(ii) Hence find an expression for $\sin 5\theta$ entirely in terms of $\sin\theta$. 3

[END OF QUESTION PAPER]

SECTION I

Attempt ALL questions in this Section.

Marks

1. Differentiate and simplify where possible

(*a*) $f(x) = \ln(1 + x^2)$, 2

(*b*) $g(x) = \dfrac{2 + \sin x}{2 + \cos x}$. 3

2. A boy found that when he played his sister at snooker he had a probability of 0·4 of winning any one game and 0·6 of losing.
He plays three games against his sister.

(*a*) Draw a tree diagram showing all possible results and their probabilities, assuming that each game is independent. 3

(*b*) What is the probability of the boy winning two of the games and losing the other? 2

3. Express the complex number $\dfrac{1 + 3i}{1 - 2i}$ in the form $x + iy$, where x and y are real numbers. 3
Determine the modulus and the argument of this complex number. 3

4. The numbers of half-days the 27 pupils in a certain secondary school class were absent during the last school year are given below. Make a stem-and-leaf plot of these numbers and hence find the median. 4

24	0	12	8	3	23	44	2	6	32
26	40	0	10	2	62	0	6	10	20
36	2	38	13	20	2	33			

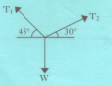
5. The function f is defined by

$$f(x) = x^3e^{-x}.$$

Find the stationary values of f and determine their nature. 5
State the interval on which the gradient of the function is negative. 1

Marks

6. A body of mass 4 kilograms is attached to two wires which suspend the body in a vertical plane. One wire makes an angle of 45° with the horizontal and the other wire makes an angle of 30° with the horizontal.



Calculate the weight of the body and the magnitude of the tensions in the two wires. 5
[Take the magnitude of the acceleration due to gravity near the surface of the Earth as 9·8 m s^{-2}.]

7. Use the method of integration by parts to evaluate

$$\int_0^1 xe^{-2x}\,dx.$$ 5

8. A stone is thrown horizontally from a point 44 metres above a horizontal plane. The stone hits the plane 39 metres from the point vertically below the point of projection.
Find the time of flight and the initial speed of the stone. 5
[Assume that there is no air resistance and take the magnitude of the acceleration due to gravity near the surface of the Earth as 9·8 m s^{-2}.]

9. Evaluate $\displaystyle\int_0^2 \frac{1}{4 + x^2}\,dx$. 3

Calculate the approximation to this definite integral obtained by applying the composite trapezium rule with four sub-intervals. 3
Use the above results to obtain an estimate for π. 1

10. Express $\dfrac{1}{(x + 1)(x + 2)}$ in partial fractions. 2

Hence find a simple expression for the sum of the series

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \ldots + \frac{1}{(n + 1)(n + 2)}$$

in terms of n. 3
Evaluate

$$\sum_{k=1}^{\infty} \frac{1}{(k + 1)(k + 2)}.$$ 1

[Turn over

Marks

11. Apply the recurrence relation

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right)$$

with $x_0 = 2$ to calculate x_1, x_2 and x_3. 2
Find the fixed points of this recurrence relation. 3
Suggest an initial value to generate a sequence converging to the negative fixed point. 1

SECTION II

Attempt ALL questions in this Section.

Marks

12. When processing the results of a survey into preferences for flavours of ice-cream, it was found that one of the questions had been phrased ambiguously and an unknown number k of responses had been made incorrectly. As a consequence, all that could be said about the numbers preferring chocolate, vanilla or strawberry (c, v, s respectively) was that they satisfied the following three equations.

$$\begin{aligned} c - v + s &= 90 \\ 2c + 3v + 5s &= 280 \\ 3c + 4s + (k + 3)s &= 362 \end{aligned}$$

(*a*) Use Gaussian elimination to obtain an expression for s in terms of k. 5

(*b*) Given that k and s are both positive integers, find what possible values k can have. 2

(*c*) For each of these values of k, calculate the corresponding values of c, v and s. 2

(*d*) Describe **briefly** why the results of the survey, in spite of the ambiguity in the phrasing of one of the questions, indicate which is the most popular flavour of ice-cream. 1

13. The function f is defined by

$$f(x) = 2x - x^2 \qquad \text{for} \qquad 1 \leqslant x \leqslant 3.$$

(*a*) Sketch the graph of f and state the range of f. 3

(*b*) Sketch the graph of the inverse function f^{-1} and state the domain of f^{-1}. 3

(*c*) Obtain a formula for the inverse function f^{-1}. 4

14. (*a*) Express $\dfrac{1}{x - x^2}$ in partial fractions. 2

(*b*) The spread of a disease in a large population can be modelled by means of a differential equation. The proportion x of the population infected with the disease after t days satisfies

$$\frac{dx}{dt} = \frac{1}{2^7}x - \frac{1}{2}x^2 \qquad \text{for } t \geqslant 0.$$

(i) Given that $x = \dfrac{1}{500}$ when $t = 0$, find x in terms of t. 7

(ii) Verify that about 6% of the population was infected after seven days. 1

(iii) How long will it take for 25% of the population to become infected? 1

[Turn over