SECTION I

Attempt ALL questions in this Section.

Marks

1. Given that $z = 2 + 3i$, plot on an Argand diagram the points that represent the complex numbers z, $\bar{z}$ and z^2, where $\bar{z}$ is the complex conjugate of z. **5**

2. A function f is defined by

$$f(x) = \frac{2x}{x-1}, \; x \neq 1.$$

Find $f'(x)$ and deduce that the derivative is always negative. **3**

3. A stone falls from a height of 10 metres above the ground under a constant acceleration, due to gravity, of magnitude 9·8 metres per second per second. Given that it is initially at rest, calculate how long it will take to reach the ground. **4**

4. Write the complex number $z = \sqrt{2}\,(1 + i)$ in polar form and verify that z satisfies the equation

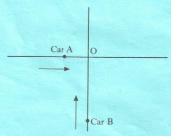
$$z^4 + 16 = 0\,.$$ **5**

5. Express $\frac{5x-4}{x^2-4x}$ in partial fractions. **3**

Hence find the indefinite integral

$$\int \frac{5x-4}{x^2-4x}\,dx.$$ **3**

6. Two cars A and B are approaching a right-angled intersection O at constant speeds, as shown in the diagram.



At $t = 0$, car A is 100 metres from O and is travelling at 10 metres per second, while at the same instant car B is 300 metres from O and is travelling at 20 metres per second.

Show that the subsequent distance d between the two cars at time t is given by

$$d^2 = 100\,(5t^2 - 140t + 1000).$$ **5**

Hence find the time at which the cars are closest. **2**

Marks

7. The number of strands of bacteria $B(t)$ present in a culture after t days of growth is assumed to be increasing at a rate proportional to the number of strands present. Write down a differential equation for B and solve it to find B in terms of t. **4**

Given that the number of strands observed after 1 day is 502 and after 4 days is 1833, find the number of strands initially present. **3**

8. Given that

$$y = e^{-x}\sin x\,,$$

find the stationary values of y for $0 \leqslant x \leqslant 2\pi$ and determine for each stationary value whether it is a local maximum or a local minimum. **7**

9. A newsagent, on reviewing her sales, decides to mount an advertising campaign in her neighbourhood. The frequency distribution of the newspapers sold in the first 140 days after the advertising campaign is given below.

Number of newspapers sold per day	65	76	82	86	89	91	95	97	104	110	117
Number of days	3	4	10	17	18	24	21	14	16	8	5

Calculate the mean of the number of newspapers sold per day. **1**

The standard deviation associated with the above data is 9·8.

The sales' records of the newsagent show that the mean and the standard deviation of the number of newspapers sold per day during the corresponding 140 days in the previous year were 83·0 and 13·8 respectively.

Comment on the effect of the advertising campaign. **2**

10. The probability of being caught in a traffic jam in the morning rush hour is 0·6 on any particular weekday.

Find

(*a*) the probability that the journey is free from traffic jams for 3 consecutive weekdays; **1**

(*b*) the probability that there is a traffic jam on at least 2 out of 3 consecutive weekdays. **3**

You may assume that occurrences of traffic jams on different days are independent of each other.

11. Use Gaussian elimination to solve the equations

$$\begin{aligned} x + y &= 4\,, \\ x + 2y - z &= 5\,, \\ 3y - 2z &= 6\,. \end{aligned}$$ **5**

(You must show your working.)

[Turn over

Marks

12. A body is acted upon by two forces

$$\mathbf{F}_1 = 2\mathbf{i} + 3\mathbf{j} \;, \quad \mathbf{F}_2 = -4\mathbf{i} + 7\mathbf{j} \;,$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the directions of increasing x and y respectively. Find the magnitude of the resultant and the angle between the resultant and $\mathbf{F}_1$. **4**

SECTION II

Attempt ALL questions in this Section.

Marks

13. Find the stationary points of the function f given by

$$f(x) = x^3 - 3x + 1\,, \quad x \in \mathbf{R}$$

and determine their nature. Sketch the graph of this function, indicating the approximate position of each of its zeros.

Justify your answer. **5**

Newton's method for improving an approximation to a root of the equation $f(x) = 0$ uses the recurrence relation

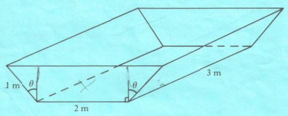
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\,.$$

For the function f defined above, obtain an expression for x_{n+1} in terms of x_n. **1**

Use Newton's method to find, correct to three decimal places, the negative root of

$$x^3 - 3x + 1 = 0\,.$$ **4**

14. A plant container is moulded from sheet plastic into the shape shown below.



Express the volume V cubic metres of the container in terms of the angle θ, where $0 \leqslant \theta < \pi/2$. **3**

Find the value of θ for which $\frac{dV}{d\theta} = 0$ and hence find the maximum volume of such a container.

Justify your answer. **7**

[Turn over

Marks

15. By writing

$$(k+1)^2 - k^2 = 2k + 1,$$

show that

$$\sum_{k=1}^{n} \{(k+1)^2 - k^2\} = 2\sum_{k=1}^{n} k + n.$$ **1**

Hence show that

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}\,.$$ **3**

Extend this idea by considering $(k+1)^3 - k^3$ to show that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}\,.$$ **6**

16. A function F is defined by the definite integral

$$F(a) = \int_0^1 x^{a-1} e^{-x}\,dx\,, \quad a > 0\,.$$

(*a*) Use integration by parts to show that

$$F(a) = \frac{1}{a}\,\{e^{-1} + F(a+1)\}.$$ **4**

(*b*) Use the composite trapezium rule with sub-interval length $h = 0{\cdot}25$ to obtain an approximation for $F(\tfrac{3}{2})$, rounded to three decimal places. **4**

(*c*) From the results obtained in (*a*) and (*b*), find an approximation for $F(\tfrac{1}{2})$, rounded to two decimal places. **2**

[END OF QUESTION PAPER]