Completing the Square

LI

- Write any quadratic expression in the form $p(x + q)^2 + r$.

SC

- Factorise.
- Simplify expressions.

Any quadratic expression can be written in the form :

$$p(x + q)^2 + r$$

This is called completing the square.

Example 1 (x^2 coefficient $= 1$; x coefficient even)

Express $x^2 + 6x + 5$ in the form $p(x + q)^2 + r$.

$$x^2 + 6x + 5$$

$$= x^2 \boxed{+ 6}x + 3^2 - 3^2 + 5$$

half the x-coefficient → square it

subtract it to keep expression the same (always subtract)

$$= (x \boxed{+ 3})^2 - 9 + 5$$

$$= \boxed{(x + 3)^2 - 4}$$

Example 2 (x^2 coefficient $= 1$; x coefficient odd)

Express $x^2 - 7x + 2$ in the form $p(x + q)^2 + r$.

$$x^2 - 7x + 2$$

$$= x^2 - 7x + (-7/2)^2 - (-7/2)^2 + 2$$

$$= (x - 7/2)^2 - 49/4 + 2$$

$$= \boxed{(x - 7/2)^2 - 41/4}$$

Example 3 (x^2 coefficient $\neq$ 1, but still positive)

Express $3x^2 - 12x - 7$ in the form $p(x + q)^2 + r$.

$$3x^2 - 12x - 7$$

$$= 3(x^2 - 4x) - 7$$

$$= 3\left[x^2 - 4x + (-2)^2 - (-2)^2\right] - 7$$

$$= 3\left[(x - 2)^2 - 4\right] - 7$$

$$= 3(x - 2)^2 - 12 - 7$$

$$= \boxed{3(x - 2)^2 - 19}$$

Example 4 (x^2 coefficient negative)

Express $12 - 8x - x^2$ in the form $p(x + q)^2 + r$.

$$12 - 8x - x^2$$

$$= -x^2 - 8x + 12$$

$$= -(x^2 + 8x) + 12$$

$$= -\left[x^2 + 8x + 4^2 - 4^2\right] + 12$$

$$= -\left[(x + 4)^2 - 16\right] + 12$$

$$= -(x + 4)^2 + 16 + 12$$

$$= \boxed{-(x + 4)^2 + 28}$$

or $\boxed{28 - (x + 4)^2}$

Questions

1) Write in the form $(x + a)^2 + b$ and state the values of a and b.

a $x^2 + 10x + 3$	**b** $y^2 - 4y + 6$	**c** $t^2 + 14t - 9$	**d** $m^2 - 6m + 4$
e $w^2 - 20w + 10$	**f** $x^2 + 12x - 3$	**g** $x^2 + 8x + 1$	**h** $m^2 + 7m + 3$
i $x^2 + 3x - 1$	**j** $a^2 - 4a - 2$	**k** $w^2 - 18w + 5$	**l** $t^2 + 9t - 3$

2) Complete the square.

a $m^2 + 2m$	**b** $t^2 - 10t$	**c** $x^2 + 12x$	**d** $y^2 - 8y + 4$
e $a^2 - 4a - 3$	**f** $t^2 + 22t - 15$	**g** $p^2 + 16p - 7$	**h** $m^2 + 2m + 7$
i $y^2 + 10y - 5$	**j** $y^2 - 5y + 3$	**k** $a^2 - a + 4$	**l** $x^2 + 7x - 2$

3) Complete the square, leaving your answers in the form $a(x + p)^2 + q$.

a $4x^2 + 16x + 3$	**b** $2y^2 + 12y - 3$	**c** $5t^2 - 30t - 8$
d $-m^2 + 6m + 2$	**e** $6w^2 + 12w - 4$	**f** $3t^2 + 12t - 3$

Answers

1)	a	$(x+5)^2 - 22; a = 5; b = -22$	2)	a	$(m+1)^2 - 1$	3)	a $4(x+2)^2 - 13$
	b	$(y-2)^2 + 2; a = -2; b = 2$		b	$(t-5)^2 - 25$		b $2(y+3)^2 - 21$
	c	$(t+7)^2 - 58; a = 7; b = -58$		c	$(x+6)^2 - 36$		c $5(t-3)^2 - 53$
	d	$(m-3)^2 - 5; a = -3; b = -5$		d	$(y-4)^2 - 12$		d $11 - (m-3)^2$
	e	$(w-10)^2 - 90; a = -10; b = -90$		e	$(a-2)^2 - 7$		e $6(w+1)^2 - 10$
	f	$(x+6)^2 - 39; a = 6; b = -39$		f	$(t+11)^2 - 136$		f $3(t+2)^2 - 15$
	g	$(x+4)^2 - 15; a = 4; b = -15$		g	$(p+8)^2 - 71$		
	h	$\left(m+\frac{7}{2}\right)^2 - \frac{37}{4}; a = \frac{7}{2}; b = -\frac{37}{4}$		h	$(m+1)^2 + 6$		
	i	$\left(x+\frac{3}{2}\right)^2 - \frac{13}{4}; a = \frac{3}{2}; b = -\frac{13}{4}$		i	$(y+5)^2 - 30$		
	j	$(a-2)^2 - 6; a = -2; b = -6$		j	$\left(y-\frac{5}{2}\right)^2 - \frac{13}{4}$		
	k	$(w-9)^2 - 76; a = -9; b = -76$		k	$\left(a-\frac{1}{2}\right)^2 + \frac{15}{4}$		
	l	$\left(t+\frac{9}{2}\right)^2 - \frac{93}{4}; a = \frac{9}{2}; b = -\frac{93}{4}$		l	$\left(x+\frac{7}{2}\right)^2 - \frac{57}{4}$		