SCOTTISH
QUALIFICATIONS
AUTHORITY

2004 Applied Mathematics

Advanced Higher – Section G

Finalised Marking Instructions

Advanced Higher Applied Mathematics 2004
Solutions for Section G (Mechanics 1)

G1.

$\mathbf{r}(t) = (2t^2 - t)\mathbf{i} - (3t + 1)\mathbf{j}$

$\Rightarrow \mathbf{v}(t) = (4t - 1)\mathbf{i} - 3\mathbf{j}$ **1**

$\Rightarrow |\mathbf{v}(t)| = \sqrt{(4t - 1)^2 + 9}$ **1**

When the speed is 5,

$(4t - 1)^2 + 9 = 25$ **1**

$(4t - 1)^2 = 16$

$4t - 1 = \pm 4$

$t = \frac{5}{4}$ seconds (as $t > 0$). **1**

G2. (a)

$\mathbf{v}_F = 25\sqrt{2}(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$ **1**

$= 25(\mathbf{i} + \mathbf{j})$

$\mathbf{r}_F = 25t(\mathbf{i} + \mathbf{j})$ as $\mathbf{r}_F(0) = \mathbf{0}$ **1**

$\mathbf{v}_L = 20\mathbf{j}$

$\mathbf{r}_L = 20t\mathbf{j} + \mathbf{c}$

But $\mathbf{r}_L(0) = 10\mathbf{i}$ so $\mathbf{r}_L = 10\mathbf{i} + 20t\mathbf{j}$ **1**

The position of the ferry relative to the freighter is

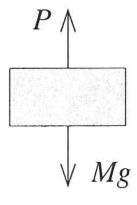
$\mathbf{r}_F - \mathbf{r}_L = (25t - 10)\mathbf{i} + 5t\mathbf{j}$ **1**

(b) When $t = 1$

$|\mathbf{r}_F - \mathbf{r}_L| = \sqrt{15^2 + 5^2}$ **1**

$= \sqrt{250} = 5\sqrt{10}$ km **1**

G3.



$P = Mg$ **1**

Combined mass $= M + 0{\cdot}01M = 1.01M$.
By Newton II

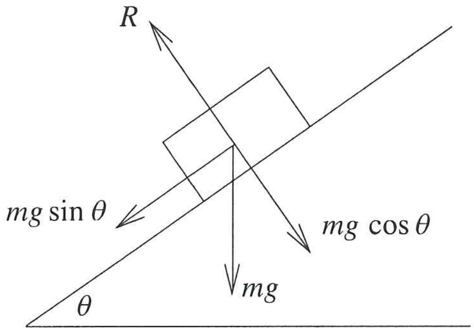
$1{\cdot}01Ma = (P + 0{\cdot}05P) - 1{\cdot}01Mg$ **1M,1**

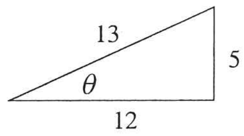
$1.01Ma = 1.05Mg - 1.01Mg$

$1.01a = 0.04g$ **1**

$a = \frac{4}{101}g\ (\approx 0.3)\ \text{m s}^{-2}$ **1**

G4.





Resolving perp. to plane: $R = mg\cos\theta$
Parallel to the plane (by Newton II)

$$ma = -\mu R - mg\sin\theta$$

$$= -\mu mg\cos\theta - mg\sin\theta \qquad \textbf{2E1}$$

$$a = -g(\mu\cos\theta + \sin\theta)$$

$$= \frac{-(5 + 12\mu)g}{13} \qquad \textbf{2E1}$$

Using $v^2 = u^2 + 2as$

$$0 = gL - \frac{2(5 + 12\mu)gL}{13} \qquad \textbf{1}$$

$$gL = \frac{2(5 + 12\mu)gL}{13}$$

$$10 + 24\mu = 13 \Rightarrow \mu = \frac{1}{8} \qquad \textbf{2E1}$$

G5. (a) The equations of motion give

$$\ddot{y} = -g \qquad \mathbf{v}(0) = V\cos\alpha\mathbf{i} + V\sin\alpha\mathbf{j}$$
$$\dot{y} = -gt + V\sin\alpha$$
$$y = V\sin\alpha t - \frac{1}{2}gt^2 \qquad \mathbf{1}$$

Maximum height when $\dot{y} = 0 \Rightarrow t = \frac{V}{g}\sin\alpha$, and so **1**

$$H = V\sin\alpha \times \frac{V}{g}\sin\alpha - \frac{1}{2}g\frac{V^2}{g^2}\sin^2\alpha \qquad \mathbf{1}$$
$$= \frac{V^2}{2g}\sin^2\alpha$$

(b) (i)

$$h = \frac{V^2}{2g}\sin^2 2\alpha$$
$$= \frac{V^2}{2g}4\sin^2\alpha\cos^2\alpha \qquad \mathbf{1}$$
$$= \frac{2V^2}{g}\sin^2\alpha(1 - \sin^2\alpha) \qquad \mathbf{1}$$
$$= 4H\left(1 - \frac{2gH}{V^2}\right) \qquad \text{since } \sin^2\alpha = \frac{2gH}{V^2} \qquad \mathbf{1}$$

(ii) Since $h = 3H$

$$3H = 4H(1 - \sin^2\alpha) \qquad \mathbf{1}$$
$$\frac{3}{4} = 1 - \sin^2\alpha$$
$$\sin^2\alpha = \frac{1}{4} \qquad \mathbf{1}$$
$$\sin\alpha = \pm\frac{1}{2} \qquad \mathbf{1}$$
$$\Rightarrow \alpha = \frac{\pi}{6} \text{ and so } 2\alpha = \frac{\pi}{3} \qquad \mathbf{1}$$