SCOTTISH
QUALIFICATIONS
AUTHORITY

2004 Applied Mathematics

Advanced Higher – Section F

Finalised Marking Instructions

Advanced Higher Applied Mathematics 2004
Solutions for Section F (Numerical Analysis 1)

F1. $f(x) = \ln(2-x)$ $f'(x) = \dfrac{-1}{(2-x)}$ $f''(x) = \dfrac{-1}{(2-x)^2}$ $f'''(x) = \dfrac{-2}{(2-x)^3}$ **1**

Taylor polynomial is

$$p(1+h) = \ln 1 - h - \frac{h^2}{2} - \frac{2h^3}{6}$$

$$= -h - \frac{h^2}{2} - \frac{h^3}{3}$$ **1**

For $\ln 1{\cdot}1$, $h = -0{\cdot}1$ and $p(0{\cdot}9) = 0{\cdot}1 - 0{\cdot}005 + 0{\cdot}00033 = 0{\cdot}0953$. **1,1**

$$p(a+h) = \ln(2-a) - \frac{1}{2-a}h$$ **1**

Hence expect $f(x)$ to be more sensitive in I_2 since coefficient of h is much larger. **1**

F2. $L(2{\cdot}5)$

$$= \frac{(2{\cdot}5-1{\cdot}5)(2{\cdot}5-3{\cdot}0)(2{\cdot}5-4{\cdot}5)}{(0{\cdot}5-1{\cdot}5)(0{\cdot}5-3{\cdot}0)(0{\cdot}5-4{\cdot}5)}1{\cdot}737 + \frac{(2{\cdot}5-0{\cdot}5)(2{\cdot}5-3{\cdot}0)(2{\cdot}5-4{\cdot}5)}{(1{\cdot}5-0{\cdot}5)(1{\cdot}5-3{\cdot}0)(1{\cdot}5-4{\cdot}5)}2{\cdot}412$$

$$+ \frac{(2{\cdot}5-0{\cdot}5)(2{\cdot}5-1{\cdot}5)(2{\cdot}5-4{\cdot}5)}{(3{\cdot}0-0{\cdot}5)(3{\cdot}0-1{\cdot}5)(3{\cdot}0-4{\cdot}5)}3{\cdot}284 + \frac{(2{\cdot}5-0{\cdot}5)(2{\cdot}5-1{\cdot}5)(2{\cdot}5-3{\cdot}0)}{(4{\cdot}5-0{\cdot}5)(4{\cdot}5-1{\cdot}5)(4{\cdot}5-3{\cdot}0)}2{\cdot}797$$ **2**

$$= -\frac{1\times 1{\cdot}737}{10} + \frac{2\times 2{\cdot}412}{4{\cdot}5} + \frac{4\times 3{\cdot}284}{2{\cdot}5\times 2{\cdot}25} - \frac{1\times 2{\cdot}797}{18}$$

$$= -0{\cdot}1737 + 1{\cdot}0720 + 2{\cdot}3353 - 0{\cdot}1554 = 3{\cdot}078$$ **2**

F3. $\Delta^2 f_0 = \Delta f_1 - \Delta f_0 = (f_2 - f_1) - (f_1 - f_0) = f_2 - 2f_1 + f_0$ **1**

Maximum rounding error $= \varepsilon + 2\varepsilon + \varepsilon = 4\varepsilon$. **1**

$\Delta^2 f_0 = 2{\cdot}618 - 2\times 2{\cdot}369 + 2{\cdot}124 = 0{\cdot}004$ **1**

and $4\varepsilon = 4 \times 0{\cdot}0005 = 0{\cdot}002$. **1**

$\Delta^2 f_0$ appears to be significantly different from 0. **1**

F4. (a) Difference table is:

i	x	$f(x)$	diff1	diff2	diff3
0	0	1·023	352	−95	3
1	0·5	1·375	257	−92	−4
2	1	1·632	165	−96	
3	1·5	1·797	69		
4	2	1·866			

2

(b) $p = 0{\cdot}3$

$$f(0{\cdot}65) = 1{\cdot}375 + 0{\cdot}3(0{\cdot}257) + \frac{(0{\cdot}3)(-0{\cdot}7)}{2}(-0{\cdot}092)$$ **2**

$$= 1{\cdot}375 + 0{\cdot}077 + 0{\cdot}010 = 1{\cdot}462$$ **1**

(or, with $p = 1{\cdot}3$, $1{\cdot}023 + 0{\cdot}458 - 0{\cdot}019$).

F5. Trapezium rule calculation is:

x	$f(x)$	m	$mf_1(x)$	$mf_2(x)$
1	1·2690	1	1·2690	1·2690
1·25	1·1803	2		2·3606
1·5	0·9867	2	1·9734	1·9734
1·75	0·6839	2		1·3678
2	0·2749	1	0·2749	0·2749
			3·5173	7·2457

2

Hence $I_1 = 3{\cdot}5173 \times 0{\cdot}5/2 = 0{\cdot}8793$ and $I_2 = 7{\cdot}2457 \times 0{\cdot}25/2 = 0{\cdot}9057$. **1**

Difference table is:

−887	−1049
−1936	−1092
−3028	−1062
−4090	

2

$|\text{max truncation error}| = 1 \times 0{\cdot}1092/12 \approx 0{\cdot}009$ **1**

Hence $I_2 = 0{\cdot}91$ or $0{\cdot}9$. **1**

Expect to reduce error by factor 4. **1**

With n strips and step size $2h$, Taylor series for expansion of an integral I approximated by the trapezium rule is:

$I = I_n + C(2h)^2 + D(2h)^4 + \ldots = I_n + 4Ch^2 + 16Dh^4 + \ldots$ (a)

With $2n$ strips and step size h, we have: $I = I_{2n} + Ch^2 + Dh^4 + \ldots$ (b) **2**

4(b) – (a) gives $3I = 4I_{2n} - I_n - 12Dh^4 + \ldots$

i.e. $I \approx (4I_{2n} - I_n)/3 = I_{2n} + (I_{2n} - I_n)/3$ **1**

$I_R = (4 \times 0{\cdot}9057 - 0{\cdot}8793)/3 = 0{\cdot}914$ **1**