SCOTTISH
QUALIFICATIONS
AUTHORITY

2004 Applied Mathematics

Advanced Higher – Section D

Finalised Marking Instructions

Advanced Higher Applied Mathematics 2004
Solutions for Section D (Mathematics 1)

D1.

$$(4x-5y)^4 = (4x)^4 - 4\times(4x)^3(5y) + 6\times(4x)^2(5y)^2 - 4\times(4x)(5y)^3 + (5y)^4 \qquad \textbf{3E1}$$

$$= 256x^4 - 1280x^3y + 2400x^2y^2 - 2000xy^3 + 625y^4. \qquad \textbf{1}$$

When $y = \frac{1}{x}$, the term independent of x is 2400. **1**

D2.

$$y = x^2 \ln x$$

$$\frac{dy}{dx} = 2x\ln x + x^2.\frac{1}{x} = 2x\ln x + x \qquad \textbf{2E1}$$

$$\frac{d^2y}{dx^2} = 2\ln x + 2x.\frac{1}{x} + 1 = 2\ln x + 3 \qquad \textbf{2E1}$$

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2x\ln x + 3x - 2x\ln x - x = 2x \qquad \textbf{1}$$

Thus $k = 2$. **1**

D3. (a)

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & -6 \\ 3 & -1 & 1 & 7 \\ 2 & 1 & -\lambda & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & -6 \\ 0 & -4 & 7 & 25 \\ 0 & -1 & 4-\lambda & 10 \end{array}\right) \quad \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \qquad \textbf{1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & -6 \\ 0 & -4 & 7 & 25 \\ 0 & 0 & 9-4\lambda & 15 \end{array}\right) \quad \begin{array}{l} \\ \\ 4R_3 - R_2 \end{array} \qquad \textbf{1}$$

There is no solution when $\lambda = \frac{9}{4}$. **1**

(b) When $\lambda = 1$,

$$5c = 15 \Rightarrow c = 3$$

$$-4b + 21 = 25 \Rightarrow b = -1$$

$$a - 1 - 6 = -6 \Rightarrow a = 1 \qquad \textbf{2E1}$$

i.e. $a = 1, b = -1, c = 3$

D4. $x + 1 = u \Rightarrow dx = du$ — **1 for differentials**

$x = u - 1 \Rightarrow x^2 + 2 = u^2 - 2u + 3.$

$$\int \frac{x^2+2}{(x+1)^2}\,dx = \int \frac{u^2-2u+3}{u^2}\,du$$ **1 for substitution**

$$= \int 1 - \frac{2}{u} + 3u^{-2}\,du$$ **1 for simplifying**

$$= u - 2\ln|u| - 3u^{-1} + c$$ **1**

$$= x - 2\ln|x+1| - \frac{3}{x+1} + c$$ **1**

D5. (a)

$$\frac{(x-1)(x-4)}{x^2+4} = A + \frac{Bx+C}{x^2+4}$$

$$x^2 - 5x + 4 = Ax^2 + 4A + Bx + C$$ **M1**

$$A = 1,\ B = -5,\ C = 0$$ **2E1**

i.e. $f(x) = 1 - \dfrac{5x}{x^2+4}.$

(b) As $x \to \pm\infty$, $y \to 1$. **1**

[No vertical asymptotes since $x^2 + 4 \neq 0$.]

(c)

$$f(x) = 1 - \frac{5x}{x^2+4}$$

$$f'(x) = -\frac{5(x^2+4) - 10x^2}{(x^2+4)^2} = 0 \text{ at S.V.}$$ **1**

$$\Rightarrow 20 - 5x^2 = 0 \Rightarrow x = \pm 2$$ **1**

$$\Rightarrow (2, -\tfrac{1}{4}) \text{ and } (-2,\ 2\tfrac{1}{4})$$ **1**

(d) $y = 0 \Rightarrow x = 1$ or $x = 4$. **1**

$$\text{Area} = -\int_1^4 \left(1 - \frac{5x}{x^2+4}\right) dx$$ **1**

$$= -\left[x - \tfrac{5}{2}\ln(x^2+4)\right]_1^4$$ **1**

$$= -\left[4 - \tfrac{5}{2}\ln 20\right] + \left[1 - \tfrac{5}{2}\ln 5\right]$$ **1**

$$= \frac{5}{2}\ln 4 - 3 = 5\ln 2 - 3 \text{ (acceptable but not required)}$$

$$\approx 0{\cdot}47 \text{ (acceptable but not required)}$$