Section F (Numerical Analysis 1)

Answer all the questions.

Marks

Answer these questions in a separate answer book, showing clearly the section chosen.

F1. The polynomial p is the Taylor polynomial of degree three for a function f near $x = 2$. Given the function $f(x) = \sqrt{9-4x}$, where $x < 2{\cdot}25$, express $p(2 + h)$ in the form $c_0 + c_1h + c_2h^2 + c_3h^3$. **3**

Estimate the value of $f(x)$ when $x = 2{\cdot}03$ using the *second* degree approximation, giving your answer to four decimal places. **2**

Write down the principal truncation error term for this second degree approximation and calculate its value. Hence state whether the second degree approximation is likely to be accurate to four decimal places. **2**

F2. The following data are available for a function f:

x	0·0	0·2	0·5
$f(x)$	1·306	1·102	0·741

Use the Lagrange interpolation formula to obtain a quadratic approximation to $f(x)$, simplifying your answer. **4**

F3. Derive the Newton forward difference formula of degree two to fit a polynomial through the points $(x_0, f(x_0))$, $(x_1, f(x_1))$, $(x_2, f(x_2))$. **5**

F4. The following data (accurate to the degree implied) and difference table are available for a function f.

i	x_i	$f(x_i)$	Δf	$\Delta^2 f$	$\Delta^3 f$
0	2·0	2·318	197	86	15
1	2·2	2·515	283	101	25
2	2·4	2·798	384	126	41
3	2·6	3·182	510	167	
4	2·8	3·692	677		
5	3·0	4·369			

(*a*) Calculate maximum rounding error in $\Delta^3 f_0$. **1**

(*b*) Identify the value of $\Delta^2 f_3$ in this table. **1**

(*c*) State whether or not a third degree polynomial would be a good approximation for this function. **1**

(*d*) Using the Newton forward difference formula of degree *two*, and working to three decimal places, estimate $f(2{\cdot}18)$. **2**

Marks

F5. The function f is defined by $f(x) = x^2e^{-x}$.

(*a*) Use Simpson's rule with two strips and the composite Simpson's rule with four strips to obtain two estimates I_2 and I_4 respectively for the integral $I = \int_0^1 f(x)\,dx$.

Perform the calculations using five decimal places. **4**

(*b*) It is given that for this function $f^{iv}(x) = e^{-x}(x^2 - 8x + 12)$ and that $f^{v}(x)$ has no zero on the interval [0, 1]. Use this information to obtain an estimate of the maximum truncation error in I_4. **2**

Hence state the value of I_4 to a suitable accuracy. **1**

(*c*) Establish Richardson's formula to improve the accuracy of Simpson's rule by interval halving. **3**

Use Richardson extrapolation to obtain an improved estimate for I based on the values of I_2 and I_4 obtained in the first part of the question. **1**

[END OF SECTION F]

[Turn over