Section G (Mechanics 1)

Answer all the questions.

Marks

Answer these questions in a separate answer book, showing clearly the section chosen.

Where appropriate, candidates should take the magnitude of the acceleration due to gravity as 9·8 m s^{-2}.

G1. A motorcyclist moves from rest along a straight, horizontal road, with acceleration $2t\mathbf{i}$ m s^{-2}, where $\mathbf{i}$ is the unit vector in the direction of motion, and t is the time in seconds from the start of the motion.

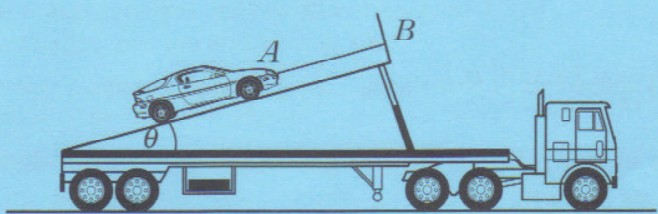
Calculate the distance travelled by the motorcyclist in the time taken for the speed to increase from 1 m s^{-1} to 9 m s^{-1}. **4**

G2. A car of mass m kilograms is travelling in a straight line along a horizontal road at constant speed U metres per second when the driver applies the brakes. The brakes cause a constant retarding force R newtons which brings the car to rest in a distance of D metres.

Find an expression for the stopping distance D in terms of m, U and R. **3**

Comment on how the stopping distance depends on the mass of the car. **1**

G3. The diagram below shows a car of mass m kilograms which is held in equilibrium on the back of a stationary lorry by means of a light inextensible chain AB which runs parallel to the sloping surface. This surface is inclined at an angle of θ to the horizontal and the coefficient of friction between the car and the surface is μ.



When $\theta = 30°$ the tension required in the chain AB to prevent the car slipping down the slope is T newtons. When θ is increased to $45°$ the tension required in AB becomes $2T$ newtons.

(*a*) When $\theta = 30°$ show that

$$T = \tfrac{1}{2}\left(1-\sqrt{3}\mu\right)mg,$$

where g m s^{-2} is the magnitude of the acceleration due to gravity. **3**

(*b*) Find an expression for T in terms of m, μ, and g when $\theta = 45°$. **2**

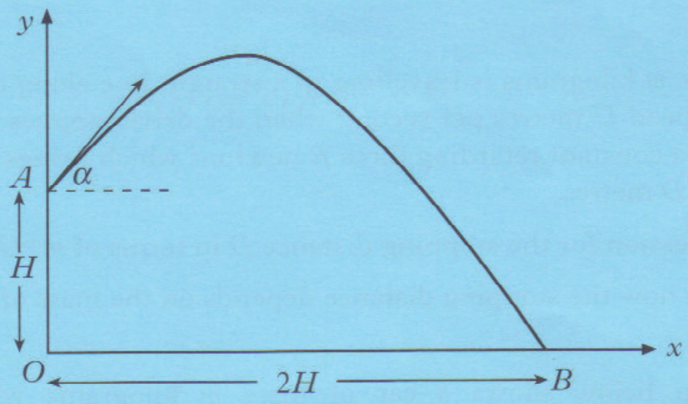
(*c*) Find the value of μ. **2**

[Turn over for Questions G4 and G5 on *Page twenty*

G4. A light aircraft is travelling due north at a constant altitude of 1 km with constant speed $100\sqrt{2}$ km/h. At 1 pm a helicopter is $50\sqrt{2}$ km due west of the aircraft, and travelling in a north easterly direction at a constant altitude of 2 km with constant speed 100 km/h.

Taking the position of the aircraft at 1 pm as the origin, and defining an appropriate set of unit vectors, find the position of the helicopter relative to the aircraft in terms of time t hours after 1 pm. **6**

G5. An artillery shell is launched from the point A, which is H metres vertically above the point O on level ground, as shown below. The shell is projected at an angle α above the horizontal, where $0 < \alpha < \frac{\pi}{2}$, with speed $\sqrt{2gH}$ metres per second, where $g\,\mathrm{m\,s^{-2}}$ is the magnitude of the acceleration due to gravity.



(*a*) Show that, referred to the axes shown, the equation of the trajectory of the shell is

$$y = H + x\tan\alpha - \frac{(1+\tan^2\alpha)}{4H}x^2.$$ **4**

[Note that $\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$.]

(*b*) The shell lands at the point B on the ground, a horizontal distance of $2H$ metres from O, as shown.

Show that $\tan\alpha = 2$. **3**

(*c*) Show further that the maximum height above the ground attained by the shell is $\frac{9}{5}H$ metres. **4**

[END OF SECTION G]

[END OF QUESTION PAPER]