Section F (Numerical Analysis 1)

Answer all the questions. *Marks*

Answer these questions in a separate answer book, showing clearly the section chosen.

F1. The function f is defined for $x > 0{\cdot}8$ by $f(x) = \dfrac{1}{5x-4}$.

The polynomial p is the Taylor polynomial of degree two for the function f near $x = 1$. Express $p(1+h)$ in the form $c_0 + c_1h + c_2h^2$. **3**

Use this polynomial to estimate the value of $f(0{\cdot}99)$. **2**

State, with a reason, whether or not $f(x)$ is sensitive to small changes in x in the neighbourhood of $x = 1$. **1**

F2. The following data are available for a function f:

x	0	2	5
$f(x)$	1·3271	1·5238	1·8516

Use the quadratic Lagrange interpolation formula to estimate $f(3)$. **3**

F3. In the usual notation for forward differences of function values $f(x)$ tabulated at equally spaced values of x,

$$\Delta f_i = f_{i+1} - f_i,$$

where $f_i = f(x_i)$ and $i = \ \ldots\ -2,\ -1,\ 0,\ 1,\ 2,\ \ldots$

Show that $\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$. **2**

If each value of f_i is subject to an error whose magnitude is less than or equal to ε, determine the magnitude of the maximum possible rounding error in $\Delta^3 f_0$. **1**

Rounded values of a function f are known to be $f_0 = 1{\cdot}311$, $f_1 = 1{\cdot}416$, $f_2 = 1{\cdot}532$, $f_3 = 1{\cdot}658$. Obtain $\Delta^3 f_0$ and the magnitude of the maximum rounding error in $\Delta^3 f_0$. **2**

Hence state whether or not this third difference appears to be significantly different from zero. **1**

F4. The following data (accurate to the degree implied) are available for a function f:

x	1·0	1·1	1·2	1·3	1·4	1·5
$f(x)$	1·263	1·456	1·696	1·991	2·351	2·782

(*a*) Construct a difference table of fourth order for the data. **3**

(*b*) Using the Newton forward difference formula of degree three, and working to three decimal places, obtain an approximation to $f(1{\cdot}18)$. **3**

[Turn over

Marks

F5. (*a*) Using a Taylor polynomial of degree two, or otherwise, derive the trapezium rule over a single strip and the corresponding principal error term. **5**

(*b*) Use the composite trapezium rule with four strips to obtain an estimate for the integral

$$\int_1^{1\cdot4} x^2 \ln x \, dx.$$

Perform the calculations using four decimal places. **2**

(*c*) Given that for $f(x) = x^2 \ln x$, $f''(x) = 2 \ln x + 3$, obtain the maximum value of $x^2 \ln x$ on the interval $[1, 1\cdot4]$ and hence obtain an estimate of the maximum truncation error in the integral. **3**

Hence state the value of the integral to a suitable accuracy. **1**

[END OF SECTION F]