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Graphs of Related Functions - Lesson 8

Graph of the Derivative

LI

- Sketch the graph of the derivative of a function.

SC

- Identify where a function is increasing, decreasing and has stationary points.

Jan 10-12:11

Reminder on Increasing, Decreasing and Stationary Points

A function $y = f(x)$ is increasing at $x = a$ if :

$$\left(\frac{dy}{dx}\right)_{x=a} > 0$$

A function $y = f(x)$ is decreasing at $x = a$ if :

$$\left(\frac{dy}{dx}\right)_{x=a} < 0$$

An interval is a set of values (usually x)

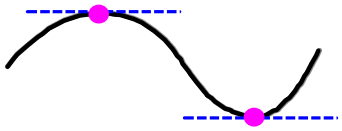
A function is increasing on an interval if the derivative is positive for every x - value in that interval

A function is decreasing on an interval if the derivative is negative for every x - value in that interval

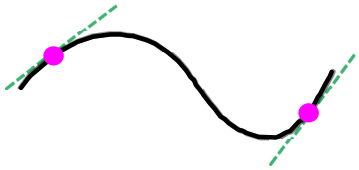
A function is stationary at $x = a$ if :

$$\left(\frac{dy}{dx}\right)_{x=a} = 0$$

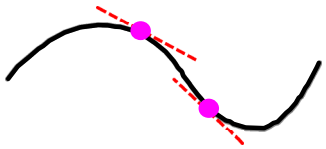
At the indicated points :



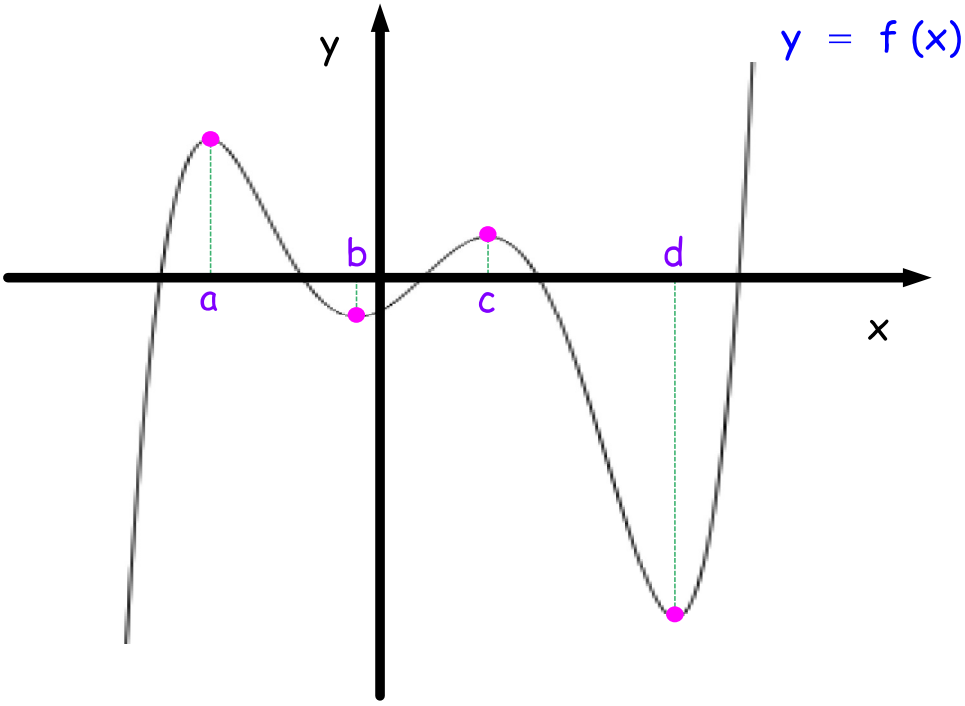
Graph is flat : 0 gradient (derivative)



Graph is increasing : + ve gradient



Graph is decreasing : – ve gradient



f is stationary when $x = a, b, c, d$.

f is increasing when $x < a$, $b < x < c$ and $x > d$.

f is decreasing when $a < x < b$ and $c < x < d$.

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Some useful guidelines

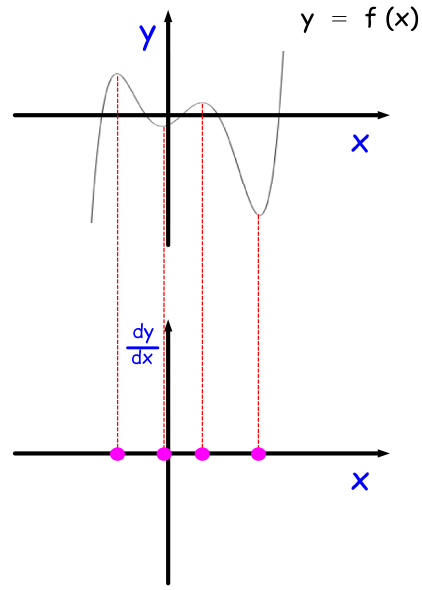
• constant function (horizontal line)	differentiates to give →	0 (horizontal line on x - axis)
• linear function (straight line)	"	constant function (horizontal line)
• quadratic function (parabola)	"	linear function (straight line)
• cubic function (cubic)	"	quadratic function (parabola)

etc.

Strategy for Sketching the Graph of the Derivative

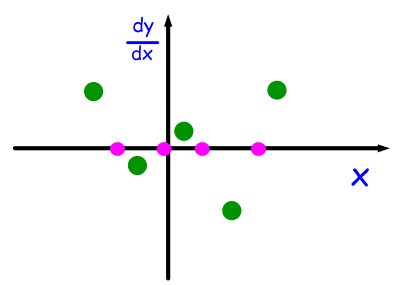
(Remember, 'gradient' means 'derivative')

- Look at where the graph of $y = f(x)$ is flat (i. e. has a 0 gradient).

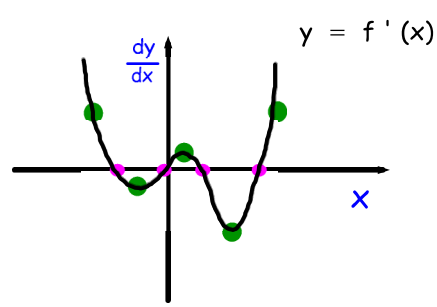


- Project these points down to the x - axis for the graph of the derivative; the graph of the derivative will cross the x - axis at these points.

- Look at regions between these x - axis points (and to the left of the leftmost point and to the right of the rightmost one) and decide if $y = f(x)$ is increasing or decreasing. If increasing, the derivative is positive and the graph of the derivative is above the x - axis; if decreasing, the graph of the derivative is below the x - axis.



- Join all the dots smoothly.



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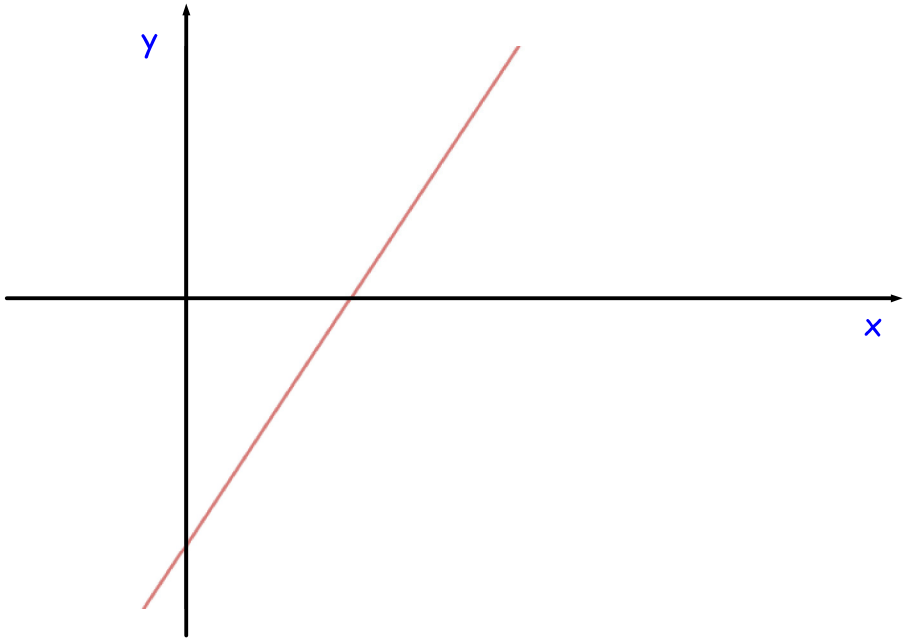
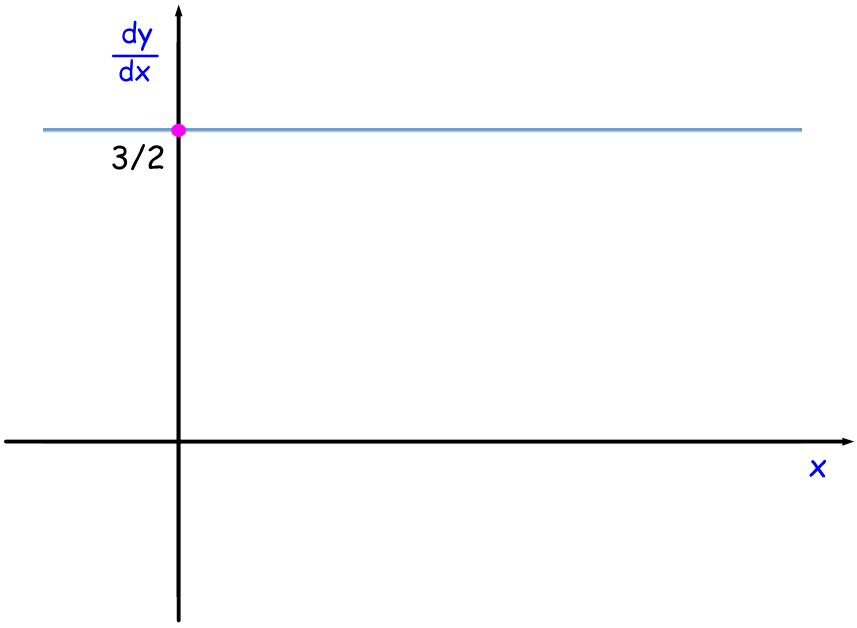
Example 1

The graph of the function $y = (3/2)x - 0.1$ is shown below.

Sketch the graph of the derivative of $f(x)$.



gradient of $f(x)$ is the same constant number
for all x - values; so, graph of $f'(x)$
is always this value



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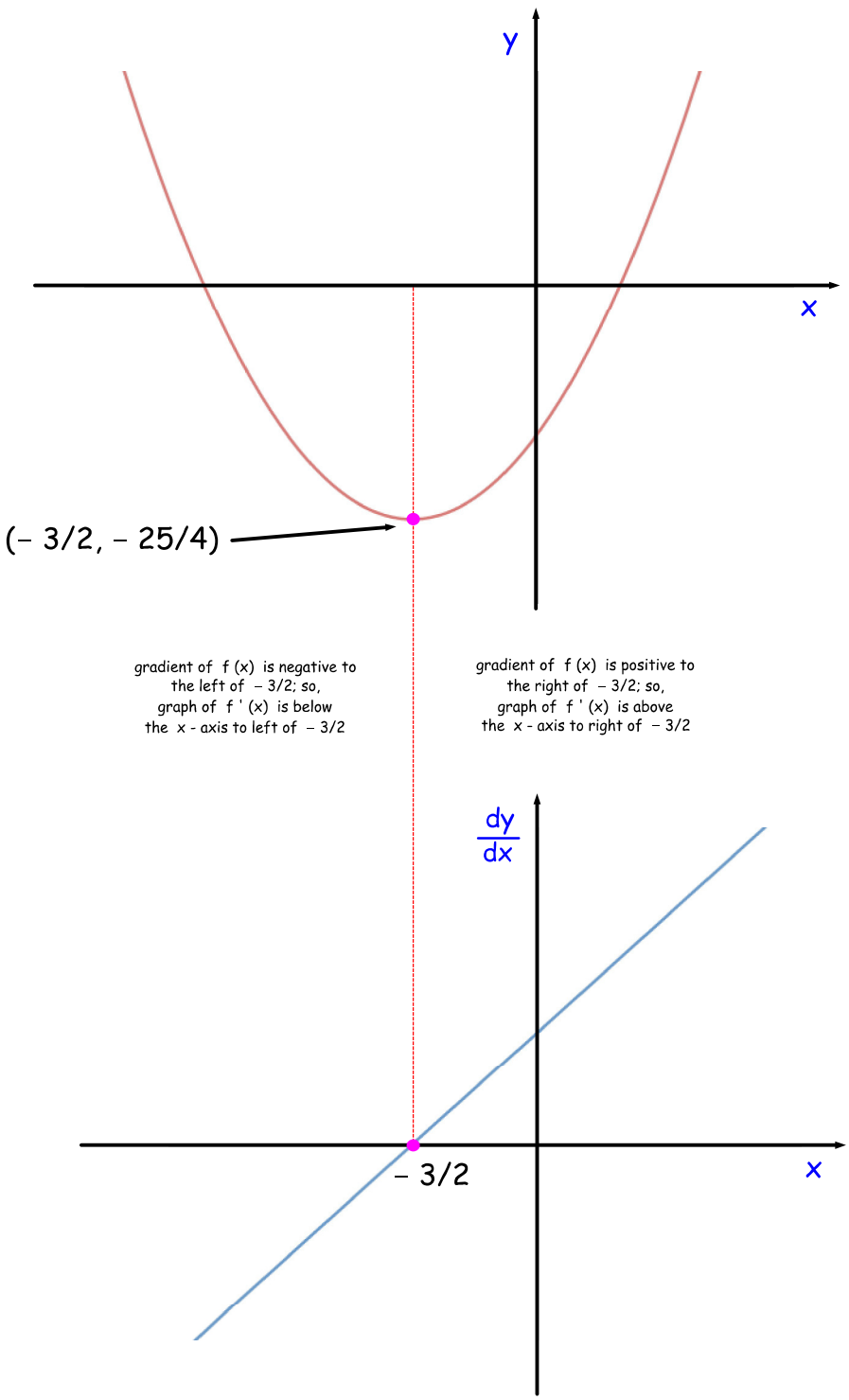
Example 2

The graph of a quadratic function $y = f(x)$ is shown below.

Sketch the graph of the derivative of $f(x)$.



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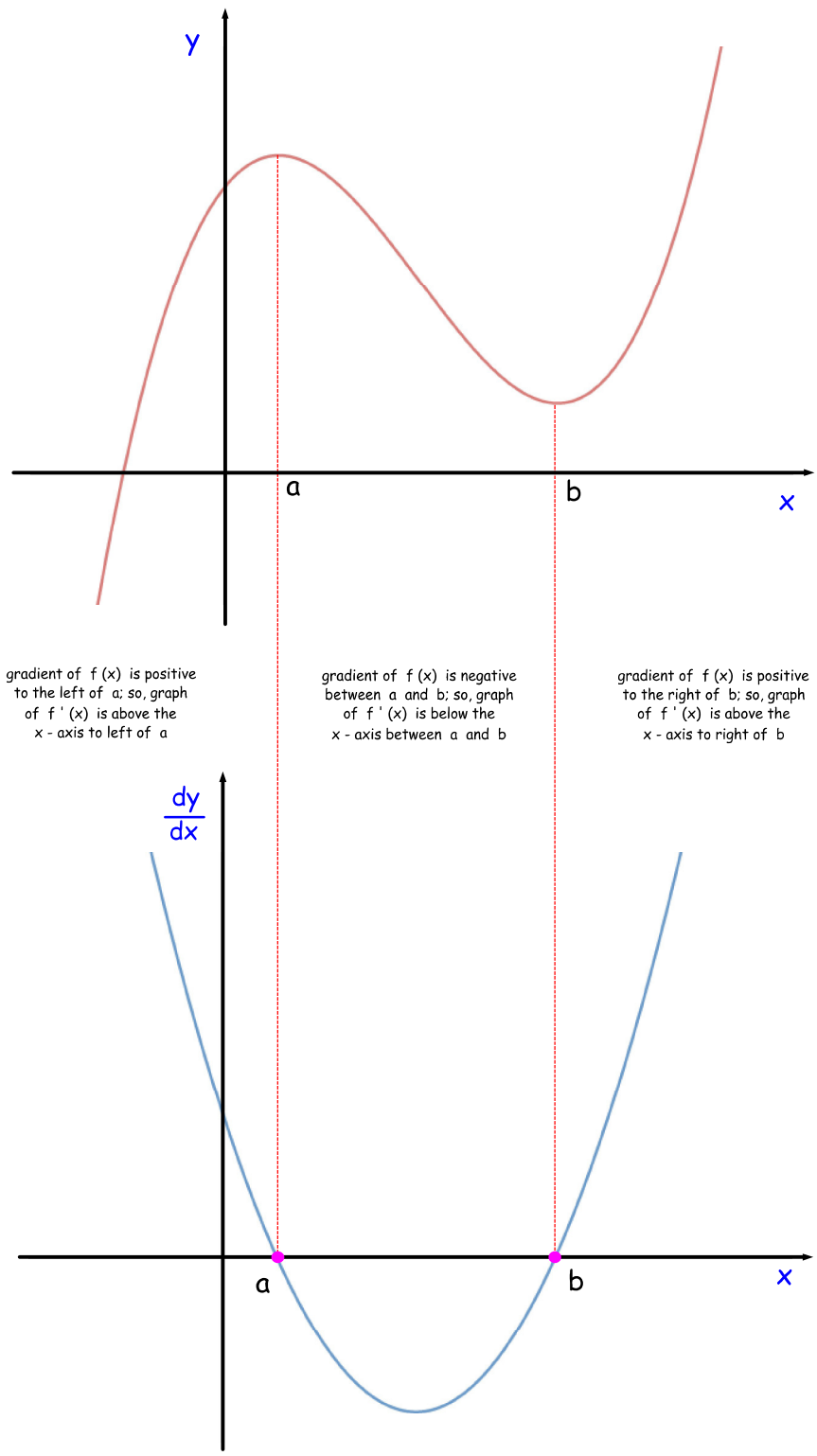
Example 3

The graph of the function $y = f(x)$ is shown below.

Sketch the graph of the derivative of $f(x)$.



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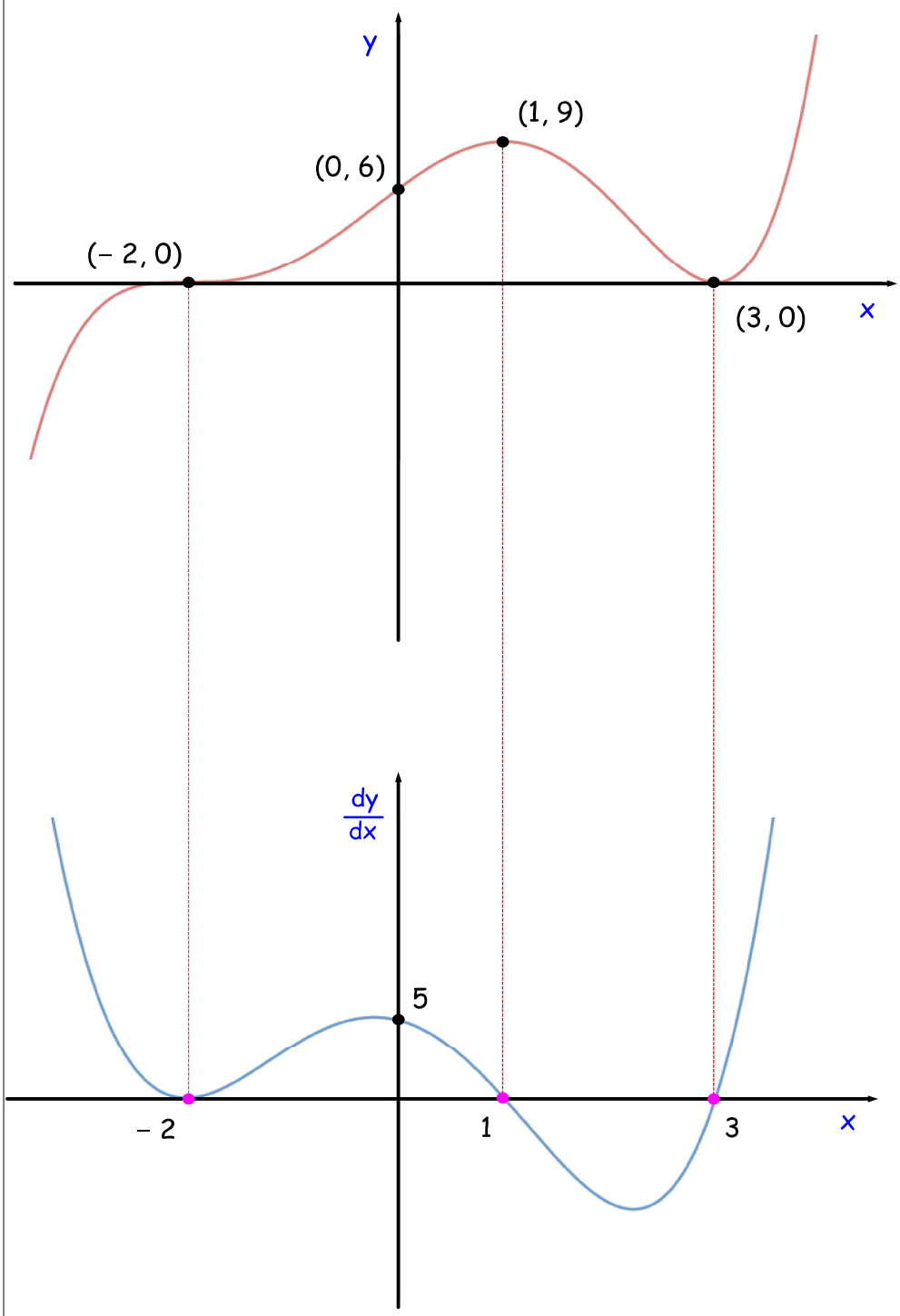
Example 4

The graph of the function $y = f(x)$ is shown below.
The gradient of the curve at the point $(0, 6)$
is 5.

Sketch the graph of the derivative of $f(x)$.



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