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Graphs of Related Functions - Lesson 7

Trigonometric Graphs

LI

- Sketch graphs of related trigonometric functions.
- Know that the order in which functions are transformed sometimes matters.

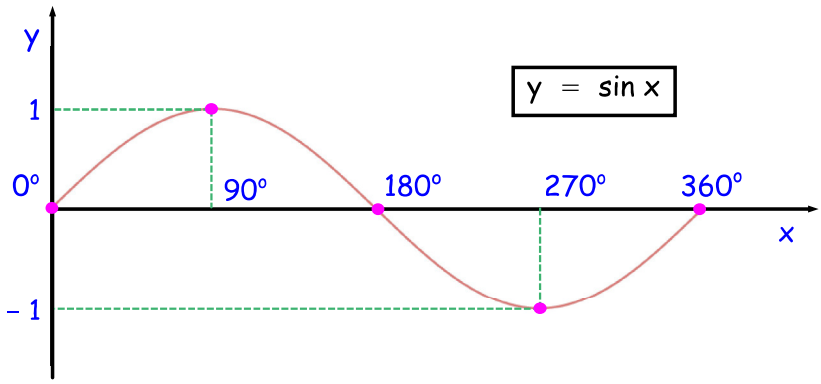
SC

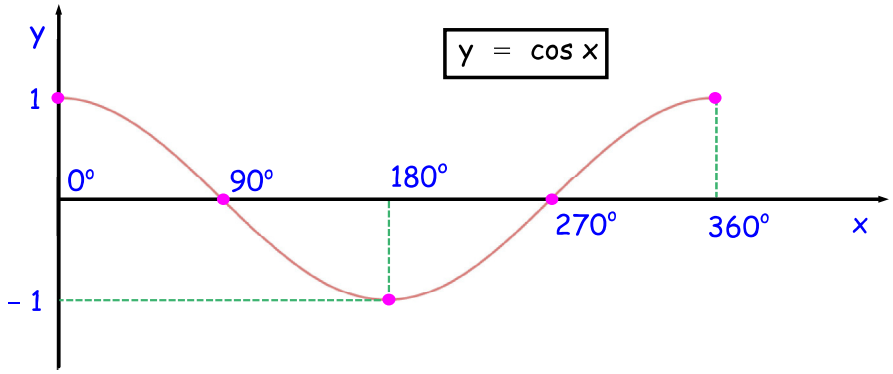
- Know the standard transformations (shifts and scalings).

Jan 10-12:11

To transform trigonometric graphs, see how the following points transform :

For Sine	For Cosine
$(0^\circ, 0)$	$(0^\circ, 1)$
$(90^\circ, 1)$	$(90^\circ, 0)$
$(180^\circ, 0)$	$(180^\circ, -1)$
$(270^\circ, -1)$	$(270^\circ, 0)$
$(360^\circ, 0)$	$(360^\circ, 1)$





For $M \sin (b\,x)^\circ$ or $M \cos (b\,x)^\circ$:

- M is the amplitude.
- b is the number of whole cycles between 0° and 360°.

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Example 1

Sketch the graph of $y = 3\cos(2x - 30)^\circ + 2$, where $0 \le x \le 180$, indicating where it cuts the y - axis.

The graph of $y = 3\cos(2x - 30)^\circ + 2$ has 2 complete cycles in $0 \le x \le 360$, so there is one complete cycle in $0 \le x \le 180$.

To see how the standard coordinates change, rewrite as,

$$\begin{aligned} y &= 3\cos(2x - 30)^\circ + 2 \\ &= 3\cos(2(x - 15))^\circ + 2 \end{aligned}$$

$(0^\circ, 1) \longrightarrow (0^\circ/2 + 15^\circ, 1 \times 3 + 2) = \underline{(15^\circ, 5)}$

$(90^\circ, 0) \longrightarrow (90^\circ/2 + 15^\circ, 0 \times 3 + 2) = \underline{(60^\circ, 2)}$

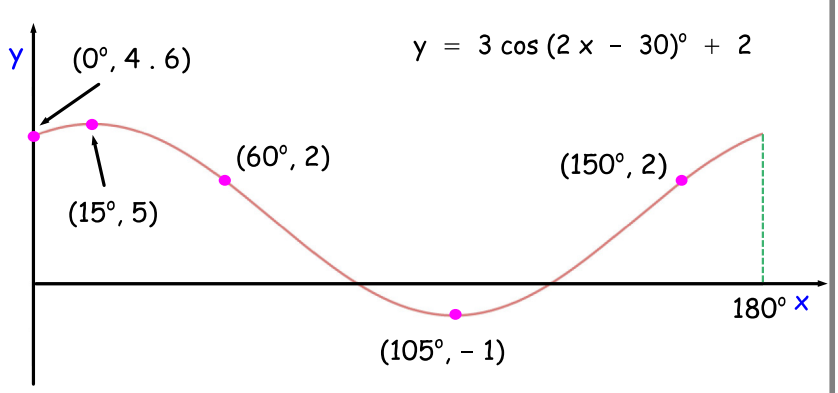
$(180^\circ, -1) \longrightarrow (90^\circ + 15^\circ, -1 \times 3 + 2) = \underline{(105^\circ, -1)}$

$(270^\circ, 0) \longrightarrow (135^\circ + 15^\circ, 0 \times 3 + 2) = \underline{(150^\circ, 2)}$

$(360^\circ, 1) \longrightarrow (180^\circ + 15^\circ, 1 \times 3 + 2) = \underline{(195^\circ, 5)}$

Since we are only considering the graph for $0 \le x \le 180$, the last transformed coordinate above will not be included in the graph.

When $x = 0$, $y = 3\cos(-30)^\circ + 2 \approx 4.6$.



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Example 2

Sketch the graph of $y = 40 \sin (10x - 60)^\circ$, where $0 \le x \le 72$, showing where the graph cuts the x - axis, and any stationary points.

The graph of $y = 40 \sin (10x - 60)^\circ$ has 10 complete cycles in $0 \le x \le 360$, so there are two complete cycles in $0 \le x \le 72$.

To see how the standard coordinates change, rewrite as,

$$y = 40 \sin (10x - 60)^\circ$$
$$= 40 \sin (10(x - 6))^\circ$$

$(0^\circ, 0) \longrightarrow (0^\circ/10 + 6^\circ, 0 \times 40) = \underline{(6^\circ, 0)}$

$(90^\circ, 1) \longrightarrow (90^\circ/10 + 6^\circ, 1 \times 40) = \underline{(15^\circ, 40)}$

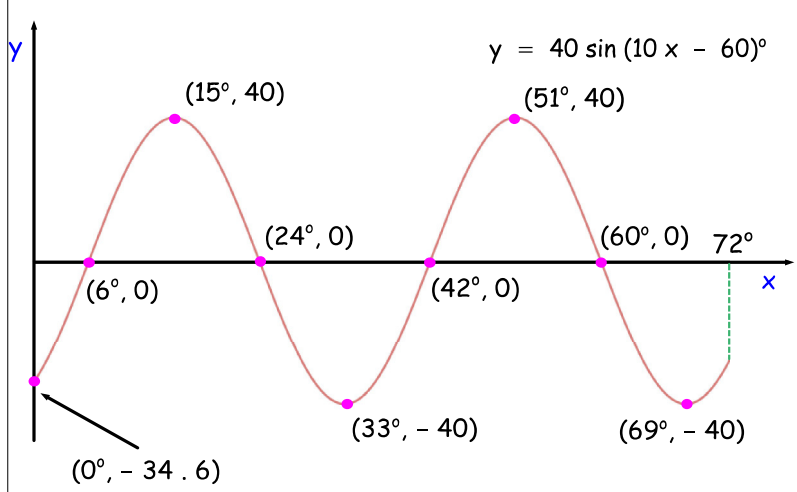
$(180^\circ, 0) \longrightarrow (180^\circ/10 + 6^\circ, 0 \times 40) = \underline{(24^\circ, 0)}$

$(270^\circ, -1) \longrightarrow (27^\circ + 6^\circ, -1 \times 40) = \underline{(33^\circ, -40)}$

$(360^\circ, 0) \longrightarrow (360^\circ/10 + 6^\circ, 0 \times 40) = \underline{(42^\circ, 0)}$

The remaining transformed coordinates can be obtained by symmetry of the graph and the above transformed coordinates. The x - coordinates increase by 9°.

When $x = 0$, $y = 40 \sin (-60)^\circ \approx -34.6$.



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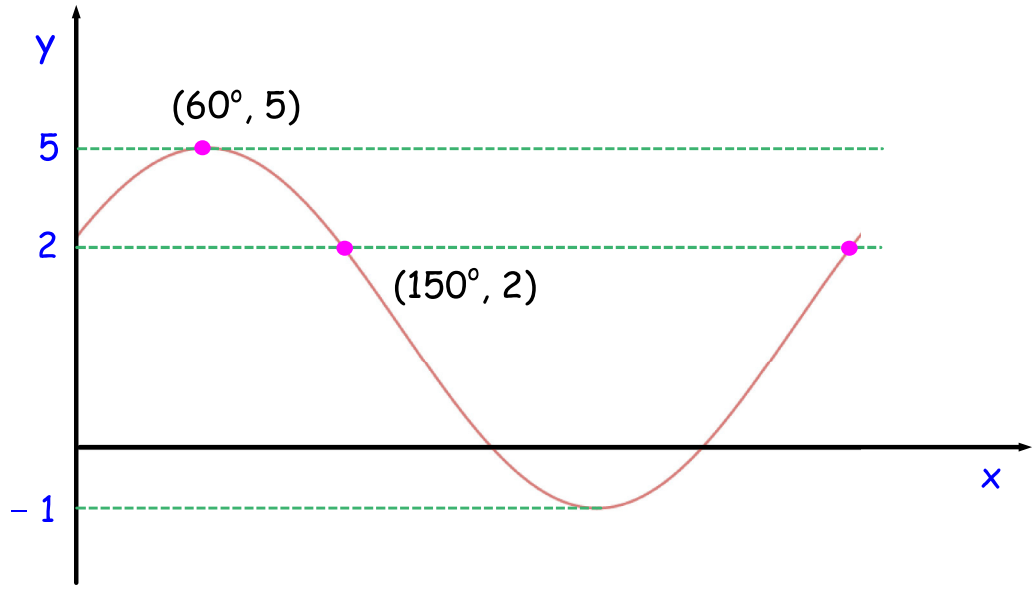
Example 3

The diagram shows the graph of
$y = P \sin (x + Q)^\circ + R$.



Find the values of P, Q and R.

The amplitude is $(5 - (-1))/2 = 3$. So, $\underline{P = 3}$.

The maximum value of $3 \sin (x + Q)^\circ$ is 3; the maximum value of $3 \sin (x + Q)^\circ + R$ is 5 (from the graph). Hence, $\underline{R = 2}$.

The maximum value of $3 \sin (x + Q)^\circ + 2$ occurs when $x = 60$ (from the graph). The maximum value of $\sin x^\circ$ occurs when $x = 90$. So, $\underline{Q = 30}$.

$$\boxed{P = 3, Q = 30, R = 2}$$

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Example 4

Given $f(x) = 2\sin(3x - 60)^\circ + 5$, where $0 \le x \le 120$, find the greatest and least values of $f(x)$ and the values of x for which they occur.

$f(x)$ is a maximum when $\sin(3x - 60)^\circ = 1$; so, the maximum value of $f(x)$ is $2(1) + 5 = 7$. This value occurs when $(3x - 60)^\circ = 90^\circ$, i.e. when $x = 50$.

$$\boxed{\text{Max. value of } f(x) = 7 \text{ occurs when } x = 50}$$

$f(x)$ is a minimum when $\sin(3x - 60)^\circ = -1$; so the minimum value of $f(x)$ is $2(-1) + 5 = 3$. This value occurs when $(3x - 60)^\circ = 270^\circ$, i.e. when $x = 110$.

$$\boxed{\text{Min. value of } f(x) = 3 \text{ occurs when } x = 110}$$

Example 5

Given $g(x) = 4 - 3\cos(x + \pi/3)$, where $0 \leq x \leq 2\pi$, find the greatest value of $g(x)$ and the value of x for which it occurs.

$g(x)$ is a maximum when $\cos(x + \pi/3) = -1$;
so, the maximum value of $g(x)$ is $4 - 3(-1) = 7$.
This value occurs when $(x + \pi/3) = \pi$,
i.e. when $x = 2\pi/3$.

$\boxed{\text{Max. value of } g(x) = 7 \text{ occurs when } x = 2\pi/3}$

Example 6

Express $\cos(30x)^\circ + \sqrt{3}\sin(30x)^\circ$ in the form $k\cos(30x - \alpha)^\circ$, where $k > 0$ and $0 \le \alpha \le 360$.

Hence sketch the graph of $y = \cos(30x)^\circ + \sqrt{3}\sin(30x)^\circ + 1$, where $0 \le x \le 12$.

Using wave function theory, we have,

$$\cos(30x)^\circ + \sqrt{3}\sin(30x)^\circ = 2\cos(30x - 60)^\circ$$

So, $y = \cos(30x)^\circ + \sqrt{3}\sin(30x)^\circ + 1$can be written as,

$$\begin{aligned} y &= 2\cos(30x - 60)^\circ + 1 \\ &= 2\cos(30(x - 2))^\circ + 1 \end{aligned}$$

The graph of $y = 2\cos(30(x - 2))^\circ + 1$ has 30 complete cycles in $0 \le x \le 360$, so there is one complete cycle in $0 \le x \le 12$.

$(0^\circ, 1) \longrightarrow (0^\circ/30 + 2^\circ, 1 \times 2 + 1) = \underline{(2^\circ, 3)}$

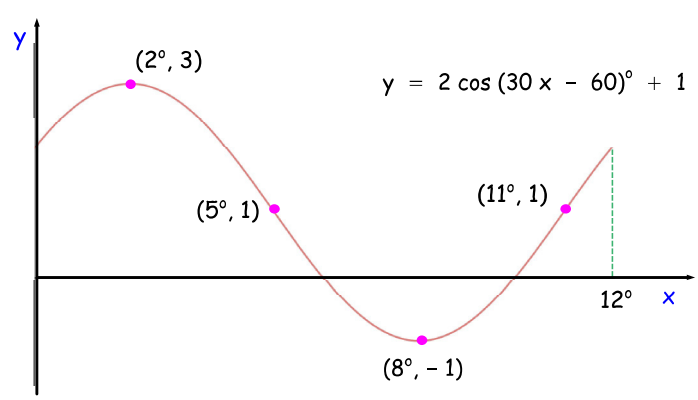
$(90^\circ, 0) \longrightarrow (90^\circ/30 + 2^\circ, 0 \times 2 + 1) = \underline{(5^\circ, 1)}$

$(180^\circ, -1) \longrightarrow (6^\circ + 2^\circ, -1 \times 2 + 1) = \underline{(8^\circ, -1)}$

$(270^\circ, 0) \longrightarrow (9^\circ + 2^\circ, 0 \times 2 + 1) = \underline{(11^\circ, 1)}$

$(360^\circ, 1) \longrightarrow (12^\circ + 2^\circ, 1 \times 2 + 1) = \underline{(14^\circ, 3)}$

Since we are only considering the graph for $0 \le x \le 12$, the last transformed coordinate above will not be included in the graph.



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