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Trigonometric Phenomena - Lesson 5

Wave Functions

LI

- Write an expression of the form $a \sin px + b \cos px$ in one of the 4 forms $k \sin (px \pm \alpha)$ or $k \cos (px \pm \alpha)$.

SC

- Addition Formulae.
- ASTC Diagram.

Jan 10-12:11

A Wave Function is an expression made up only of functions that are periodic (i.e. repeat)

In Higher Maths, this means sine and cosine

Example 1

Write $3 \sin x^\circ + 11 \cos x^\circ$ in the form $k \cos (x + \alpha)^\circ$, where $k > 0$ and $0 \le \alpha < 360$.

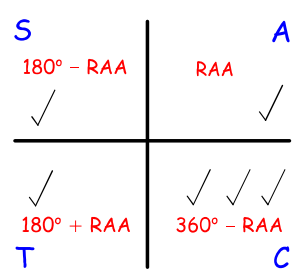
$$k \cos (x + \alpha)^\circ = k (\cos x^\circ \cos \alpha^\circ - \sin x^\circ \sin \alpha^\circ)$$
$$= (k \cos \alpha^\circ) \cos x^\circ + (-k \sin \alpha^\circ) \sin x^\circ$$
$$= 3 \sin x^\circ + 11 \cos x^\circ$$

$$\therefore \quad -k \sin \alpha^\circ = 3 \qquad \Rightarrow \qquad k \sin \alpha^\circ = -3 \qquad (1)$$
$$k \cos \alpha^\circ = 11 \qquad\qquad k \cos \alpha^\circ = 11 \qquad (2)$$

$$k = \sqrt{11^2 + (-3)^2} \qquad \tan \alpha^\circ = -\frac{3}{11} \qquad (3)$$
$$k = \sqrt{121 + 9} \qquad \tan \alpha^\circ = -0.272\ldots$$
$$\underline{k = \sqrt{130}} \qquad RAA = \tan^{-1}(0.272\ldots)$$
$$RAA = 15.3^\circ$$

$$k \sin \alpha^\circ = -3 \quad \Rightarrow \quad \sin \alpha^\circ < 0$$
$$k \cos \alpha^\circ = 11 \quad \Rightarrow \quad \cos \alpha^\circ > 0$$
$$\tan \alpha^\circ = -\frac{3}{11} \quad \Rightarrow \quad \tan \alpha^\circ < 0$$

$$\therefore \quad \alpha^\circ = 360^\circ - 15.3^\circ$$
$$\Rightarrow \quad \underline{\alpha^\circ = 344.7^\circ}$$



$$\boxed{3 \sin x^\circ + 11 \cos x^\circ = \sqrt{130} \cos (x + 344.7)^\circ}$$

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Example 2

Write $\cos x - \sin x$ in the form
$R\cos(x - \theta)$, where $R > 0$ and $0 \le \theta < 2\pi$.

$$R\cos(x - \theta) = R(\cos x \cos\theta + \sin x \sin\theta)$$
$$= (R\cos\theta)\cos x + (R\sin\theta)\sin x$$
$$= \cos x - \sin x$$

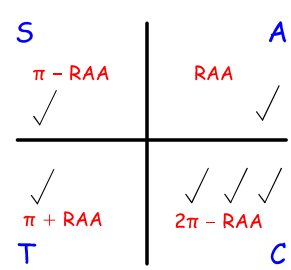
$$\therefore \quad R\sin\theta = -1 \qquad (1)$$
$$R\cos\theta = 1 \qquad (2)$$

$$k = \sqrt{(-1)^2 + 1^2}$$
$$\underline{k = \sqrt{2}}$$

$$\tan\theta = -\frac{1}{1} \qquad (3)$$
$$\tan\theta = -1$$
$$RAA = \frac{\pi}{4}$$

$$k\sin\theta = -1 \quad \Rightarrow \quad \sin\theta < 0$$
$$k\cos\theta = 1 \quad \Rightarrow \quad \cos\theta > 0$$
$$\tan\theta = -1 \quad \Rightarrow \quad \tan\theta < 0$$

$$\therefore \quad \theta = 2\pi - \frac{\pi}{4}$$
$$\Rightarrow \quad \underline{\theta = \frac{7\pi}{4}}$$



$$\boxed{\cos x - \sin x = \sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)}$$

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Example 3

Write $5\cos 2x^\circ - 12\sin 2x^\circ$ in the form $k\sin(2x - \alpha)^\circ$, where $k > 0$ and $0 \le \alpha < 360$.

$$\begin{aligned} & k\sin(2x - \alpha)^\circ \\ &= k(\sin 2x^\circ \cos\alpha^\circ - \cos 2x^\circ \sin\alpha^\circ) \\ &= (k\cos\alpha^\circ)\sin 2x^\circ + (-k\sin\alpha^\circ)\cos 2x^\circ \\ &= (-k\sin\alpha^\circ)\cos 2x^\circ + (k\cos\alpha^\circ)\sin 2x^\circ \\ &= 5\cos 2x^\circ + (-12)\sin 2x^\circ \end{aligned}$$

$$\therefore \quad \begin{aligned} -k\sin\alpha^\circ &= 5 \\ k\cos\alpha^\circ &= -12 \end{aligned} \quad \Rightarrow \quad \begin{aligned} k\sin\alpha^\circ &= -5 \quad (1) \\ k\cos\alpha^\circ &= -12 \quad (2) \end{aligned}$$

$$k = \sqrt{(-5)^2 + (-12)^2} \qquad \tan\alpha^\circ = \frac{5}{12} \quad (3)$$

$$k = \sqrt{25 + 144} \qquad \tan\alpha^\circ = 0.416\ldots$$

$$k = \sqrt{169} \qquad RAA = \tan^{-1}(0.416\ldots)$$

$$\underline{k = 13} \qquad RAA = 22.6^\circ$$

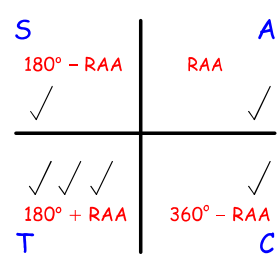
$$k\sin\alpha^\circ = -5 \quad \Rightarrow \quad \sin\alpha^\circ < 0$$

$$k\cos\alpha^\circ = -12 \quad \Rightarrow \quad \cos\alpha^\circ < 0$$

$$\tan\alpha^\circ = \frac{5}{12} \quad \Rightarrow \quad \tan\alpha^\circ > 0$$

$$\therefore \quad \alpha^\circ = 180^\circ + 22.6^\circ$$

$$\Rightarrow \quad \underline{\alpha^\circ = 202.6^\circ}$$



$$\boxed{5\cos 2x^\circ - 12\sin 2x^\circ = 13\sin(2x - 202.6)^\circ}$$

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pg. 52 Ex. 2I All Q

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