21 / 6 / 17

Unit 1 : Differential Calculus - Lesson 4

Derivatives of Exponential and Logarithmic Functions

LI

- Know the derivatives of e^x and $\ln x$.

SC

- Memorise Rules.

Jan 10-12:11

Derivatives of Exponential and Logarithmic Functions

- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} \ln x = \frac{1}{x}$

Jun 6-16:08

General Form of Derivatives - Chain Rule

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} . f'(x)$$

$$\frac{d}{dx} \ln f(x) = \frac{1}{f(x)} . f'(x)$$

Jun 8-10:14

Example 1

If $g(x) = e^{x/4}$, find $g'(x)$.

$$g(x) = e^{x/4}$$

$$\therefore \quad g'(x) = e^{x/4} \cdot \frac{d}{dx}(x/4)$$

$$\Rightarrow \quad \boxed{g'(x) = (1/4)\, e^{x/4}}$$

Mar 1-14:16

Example 2

If $f(x) = \exp(6x)$, find $f'(x)$.

$$f(x) = \exp(6x)$$

$$f(x) = e^{6x}$$

$$\therefore \quad f'(x) = e^{6x} \cdot \frac{d}{dx}(6x)$$

$$\Rightarrow \quad \boxed{\begin{aligned} f'(x) &= 6e^{6x} \\ \left(f'(x) = 6\exp(6x)\right) \end{aligned}}$$

Mar 1-14:16

Example 3

If $r(w) = \ln(w^2 + 11w)$, find $r'(w)$.

$$r(w) = \ln(w^2 + 11w)$$

$$\therefore \quad r'(w) = \frac{1}{w^2 + 11w} \cdot \frac{d}{dw}(w^2 + 11w)$$

$$\Rightarrow \quad \boxed{r'(w) = \frac{2w + 11}{w^2 + 11w}}$$

Mar 1-14:16

Example 4

If $y = \ln(\cot x)$, find y', expressing the answer in terms of $\sin x$ and $\cos x$ only.

$$y = \ln(\cot x)$$

$$\therefore \quad y' = \frac{1}{\cot x} \cdot \frac{d}{dx}(\cot x)$$

$$\Rightarrow \quad y' = -\frac{\operatorname{cosec}^2 x}{\cot x}$$

$$\Rightarrow \quad y' = -\frac{1}{\sin^2 x} \div \frac{\cos x}{\sin x}$$

$$\Rightarrow \quad y' = -\frac{1}{\sin^2 x} \times \frac{\sin x}{\cos x}$$

$$\Rightarrow \quad \boxed{y' = -\frac{1}{\sin x \cos x}}$$

Example 5

If $y = \cot(\ln x)$, find y'.

$$y = \cot(\ln x)$$

$$\therefore \quad y' = -\operatorname{cosec}^2(\ln x) \cdot \frac{d}{dx}(\ln x)$$

$$\Rightarrow \quad y' = -\operatorname{cosec}^2(\ln x) \cdot (1/x)$$

$$\Rightarrow \quad \boxed{y' = -\frac{\operatorname{cosec}^2(\ln x)}{x}}$$

Mar 1-14:16

Example 6

If $y = e^{\sin x}$, find y'.

$$y = e^{\sin x}$$

$$\therefore \quad y' = e^{\sin x} \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow \quad \boxed{y' = \cos x \, e^{\sin x}}$$

Mar 1-14:16

Example 7

If $y = \ln(\sec(x^3))$, find y'.

$$y = \ln(\sec(x^3))$$

$$\therefore \quad y' = \frac{1}{\sec(x^3)} \cdot \frac{d}{dx}(\sec(x^3))$$

$$\Rightarrow \quad y' = \frac{1}{\sec(x^3)} \cdot \sec(x^3) \cdot \tan(x^3) \cdot 3x^2$$

$$\Rightarrow \quad \boxed{y' = 3x^2 \tan(x^3)}$$

Mar 1-14:16

AH Maths - MiA (2nd Edn.)

- pg. 58-9 Ex. 4.9 Q 1 - 5.

Feb 10-15:47

Ex. 4.9

1 Find the derivative of each of these.

a e^{4x} b e^{4x+1} c e^{x^2} d e^{1-x^2} e $e^{\cos x}$

f $2e^{3x+4}$ g $3e^{\frac{x}{3}}$ h $4e^{x^3-2x}$ i $5e^{\sin x}$ j $-e^{2\cos x}$

k $\ln(x+3)$ l $\ln(3x-1)$ m $3\ln(1-2x)$ n $\ln(2x^3+5)$ o $\ln(\sin x)$

p $\ln(x+3)^2$ q $\ln\left(\frac{1}{x}\right)$ r $\sin(\ln x)$ s $(\ln(x))^3$ t $\frac{1}{\ln x}$

2 Differentiate

a $e^{\frac{1}{2x}}$ b $e^{\sin^2 x}$ c $e^{\frac{x+1}{x-1}}$ d $e^{\sin x \cos x}$ e $e^{\sec x}$

f $\ln\left(\frac{1}{x^2}\right)$ g $\ln(\sin^2 x)$ h $e^x \ln x$ i $\ln x^2 \ln(x+2)$ j $\ln(\sec x)$

3 Calculate $f'(x)$ when $f(x)$ is

a $\ln(\cos 3x)$ b $\ln(\ln(x))$ c $e^{2x+1}\ln(2x+1)$ d $3e^{\sec x}$ e e^{e^x}

4 Find $\frac{dy}{dx}$ when y is

a $(3x+1)e^{3x}$ b $\cos x e^{\cos x}$ c $e^{1-3x}\tan 2x$ d $e^{(1-\ln x)}$ e $4e^x \cot x$

5 Differentiate

a $\frac{2x}{3e^x}$ b $\frac{x+e^x}{x-e^x}$ c $\frac{e^x+e^{-x}}{e^x-e^{-x}}$ d $\frac{\ln x+\ln 2x}{e^{x-1}}$ e $\frac{(x-1)(x+2)}{e^{x-1}}$

f $\frac{\ln(x+1)}{e^x+\ln x}$ g $\frac{e^x}{\sqrt{\ln x}}$ h $\frac{\ln(x^2+2x-1)}{\sqrt{e^x}}$ i $\cos\left(\frac{\ln x}{e^x}\right)$

Answers to AH Maths (MiA), pg. 58-9, Ex. 4.9

1 **a** $4e^{4x}$ **b** $4e^{4x+1}$

c $2xe^{x^2}$ **d** $-2xe^{1-x^2}$

e $-\sin x\, e^{\cos x}$ **f** $6e^{3x+4}$

g $e^{\frac{x}{3}}$ **h** $4(3x^2-2)e^{x^3-2x}$

i $5\cos x\, e^{\sin x}$ **j** $2\sin x\, e^{2\cos x}$

k $\dfrac{1}{x+3}$ **l** $\dfrac{3}{3x-1}$

m $-\dfrac{6}{1-2x}$ **n** $\dfrac{6x^2}{2x^3+5}$

o $\dfrac{\cos x}{\sin x} = \cot x$ **p** $\dfrac{2(x+3)}{(x+3)^2} = \dfrac{2}{x+3}$

q $-\dfrac{1}{x}$ **r** $\dfrac{\cos(\ln x)}{x}$

s $\dfrac{3(\ln x)^2}{x}$ **t** $-\dfrac{1}{x(\ln x)^2}$

2 **a** $-\dfrac{1}{2x^2}e^{\frac{1}{2x}}$ **b** $2\sin x\cos x\, e^{\sin^2 x}$

c $-\dfrac{2}{(x-1)^2}e^{\frac{x+1}{x-1}}$

d $(\cos^2 x - \sin^2 x)e^{\sin x\cos x}$

e $\sec x\tan x\, e^{\sec x}$ **f** $-\dfrac{2}{x^3}x^2 = -\dfrac{2}{x}$

g $\dfrac{2\sin x\cos x}{\sin^2 x} = 2\cot x$ **h** $e^x\ln x + \dfrac{e^x}{x}$

i $\dfrac{2\ln(x+2)}{x} + \dfrac{\ln x^2}{x+2}$ **j** $\dfrac{\sec x\tan x}{\sec x} = \tan x$

3 **a** $-3\tan 3x$ **b** $\dfrac{1}{x\ln x}$

c $2e^{2x+1}\ln(2x+1) + \dfrac{2e^{2x+1}}{2x+1}$

d $3\sec x\tan x\, e^{\sec x}$ **e** $e^x \cdot e^{e^x} = e^{e^x+x}$

4 **a** $3e^{3x}(2+3x)$

b $-\sin x\, e^{\cos x}(1+\cos x)$

c $e^{1-3x}(-3\tan 2x + 2\sec^2 2x)$

d $-\dfrac{e^{1-\ln x}}{x} = -\dfrac{e^1}{xe^{\ln x}} = -\dfrac{e}{x^2}$

e $4e^x(\cot x - \operatorname{cosec}^2 x)$

5 **a** $\dfrac{2-2x}{3e^x}$ **b** $\dfrac{2e^x(x-1)}{(x-e^x)^2}$

c $\dfrac{-4}{(e^x-e^{-x})^2}$ **d** $\dfrac{2-x\ln(2x^2)}{xe^{x-1}}$

e $\dfrac{3+x-x^2}{e^{x-1}}$

f $\dfrac{(e^x+\ln x)\dfrac{1}{(x+1)} - \ln(x+1)\left(e^x+\frac{1}{x}\right)}{(e^x+\ln x)^2}$

g $\dfrac{\sqrt{\ln x}\,e^x - \dfrac{e^x}{2x\sqrt{\ln x}}}{\ln x} = \dfrac{e^x(2x\ln x - 1)}{2x(\ln x)^{\frac{3}{2}}}$

h $\dfrac{2x+2}{(x^2+2x-1)e^{\frac{x}{2}}} - \dfrac{\ln(x^2+2x-1)}{2e^{\frac{x}{2}}}$

i $-\sin\left(\dfrac{\ln x}{e^x}\right)\left(\dfrac{1-x\ln x}{x\,e^x}\right)$