Integration - Lesson 3

Integrals of Powers of Linear Functions

LI

- Integrate expressions of the form $(ax + b)^n$.

SC

- Opposite of differentiating powers of linear functions.

Applying the Chain Rule for differentiation to the function,

$$y = \frac{1}{a(n+1)}(ax+b)^{n+1}$$

(a, b and n are constants with $n \neq -1$)

we get,

$$\frac{dy}{dx} = (ax+b)^n$$

Hence,

$$\boxed{\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \quad (n \neq -1)}$$

Example 1

Integrate $y = (x + 5)^4$.

$$\int (x + 5)^4 \, dx = \frac{(x + 5)^{4+1}}{1 \,.\, (4 + 1)} + C$$

$$= \boxed{\frac{(x + 5)^5}{5} + C}$$

Example 2

Integrate $y = (3x - 7)^8$.

$$\int (3x - 7)^8 \, dx = \frac{(3x - 7)^{8+1}}{3 \,.\, (8 + 1)} + C$$

$$= \boxed{\frac{(3x - 7)^9}{27} + C}$$

Example 3

Integrate $y = \dfrac{1}{(2x - 1)^3}$.

$$\int \frac{1}{(2x - 1)^3}\, dx = \int (2x - 1)^{-3}\, dx$$

$$= \frac{(2x - 1)^{-3+1}}{2 . (-3 + 1)} + C$$

$$= -\frac{(2x - 1)^{-2}}{4} + C$$

$$\left(= -\frac{1}{4(2x - 1)^2} + C \right)$$

Example 4

Integrate $y = \sqrt[3]{2 - 5x}$.

$$\int \sqrt[3]{2 - 5x}\, dx = \int (2 - 5x)^{1/3}\, dx$$

$$= \frac{(2 - 5x)^{1/3 + 1}}{(-5)(1/3 + 1)} + C$$

$$= \frac{(2 - 5x)^{4/3}}{(-5)(4/3)} + C$$

$$= -\frac{3(2 - 5x)^{4/3}}{20} + C$$

$$\left(= -\frac{3\sqrt[3]{(2 - 5x)^4}}{20} + C \right)$$

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Questions

1 Integrate with respect to x.

e $(2x+3)^4$ **f** $(5x-2)^7$ **g** $3(4x+1)^6$

2 Integrate with respect to x.

a $(x+2)^{-3}$ **b** $(x-7)^{-2}$ **c** $8(x-5)^{-5}$ **d** $3(x+8)^{-6}$

e $(3x-1)^{-4}$ **f** $(2x+1)^{-2}$ **g** $2(4x-3)^{-5}$ **h** $7(6x-5)^{-8}$

3 Integrate with respect to x.

a $(x+1)^{\frac{1}{2}}$ **b** $(x-4)^{\frac{1}{3}}$ **c** $12(x+6)^{\frac{1}{5}}$ **d** $10(x+4)^{\frac{3}{2}}$

e $(x-2)^{-\frac{1}{2}}$ **f** $(x+1)^{-\frac{1}{3}}$ **g** $4(x-6)^{-\frac{3}{2}}$ **h** $\frac{3}{4}(x+4)^{-\frac{3}{4}}$

i $(3x+2)^{\frac{1}{2}}$ **j** $(5x-3)^{-\frac{1}{2}}$ **k** $\frac{5}{8}(6x+1)^{\frac{1}{4}}$ **l** $\frac{3}{4}(7x-4)^{-\frac{5}{2}}$

m $(2x-5)^{-\frac{2}{3}}$ **n** $\frac{1}{2}(4x-1)^{-\frac{3}{4}}$ **o** $\frac{2(8x+3)^{-\frac{5}{4}}}{3}$ **p** $(7x-1)^{\frac{5}{3}}$

4 Integrate with respect to x.

a $\frac{1}{(x-3)^2}$ **b** $\frac{3}{(x+1)^3}$ **c** $\frac{1}{5(x-2)^4}$ **d** $\frac{4}{3(x+8)^5}$

e $\frac{1}{(3x-1)^4}$ **f** $\frac{9}{(5x-3)^2}$ **g** $\frac{1}{3(2x-7)^5}$ **h** $\frac{3}{2(x+4)^6}$

5 Integrate with respect to x.

a $\sqrt{x+4}$ **b** $8\left(\sqrt{x-1}\right)$ **c** $\frac{1}{\sqrt{x-1}}$ **d** $\frac{6}{\sqrt{x+3}}$

e $\sqrt[3]{x+4}$ **f** $\sqrt[4]{(x-1)^3}$ **g** $\sqrt{(x+5)^3}$ **h** $\frac{1}{\sqrt{(x-2)^3}}$

i $8\left(\sqrt[4]{x-6}\right)$ **j** $\frac{2}{3}\left(\sqrt[5]{x+1}\right)$ **k** $\frac{5}{\left(\sqrt[3]{x-6}\right)}$ **l** $\frac{4}{5\left(\sqrt[6]{(x-1)^5}\right)}$

6 Integrate with respect to x.

a $\sqrt{3x+2}$ **b** $\sqrt{(4x-1)^3}$ **c** $3\left(\sqrt[5]{(2x-3)^2}\right)$ **d** $\frac{4}{3\left(\sqrt[6]{(7x-1)^5}\right)}$

Answers

1

e $\frac{1}{10}(2x+3)^5 + c$

f $\frac{1}{40}(5x-2)^8 + c$

g $\frac{3}{28}(4x+1)^7 + c$

2

a $-\frac{1}{2(x+2)^2} + c$

b $\frac{1}{7-x} + c$

c $-2(x-5)^{-4}$

d $-\frac{3}{5(x+8)^5} + c$

e $\frac{1}{9(1-3x)^3} + c$

f $-\frac{1}{2(2x+1)} + c$

g $-\frac{1}{8(4x-3)^4} + c$

h $\frac{1}{6(5-6x)^7} + c$

3

a $\frac{2}{3}(x+1)^{\frac{3}{2}} + c$

b $\frac{3}{4}(x-4)^{\frac{4}{3}} + c$

c $10(x+6)^{\frac{6}{5}} + c$

d $4(x+4)^{\frac{5}{2}} + c$

e $2\sqrt{x-2} + c$

f $\frac{3}{2}(x+1)^{\frac{2}{3}} + c$

g $-\frac{8}{\sqrt{x-6}} + c$

h $3(x+4)^{\frac{1}{4}} + c$

i $\frac{2}{9}(3x+2)^{\frac{3}{2}} + c$

j $\frac{2}{5}(5x-3)^{\frac{1}{2}} + c$

k $\frac{1}{12}(6x+1)^{\frac{5}{4}} + c$

l $-\frac{1}{14(7x-4)^{\frac{3}{2}}} + c$

m $\frac{3}{2}(2x-5)^{\frac{1}{3}} + c$

n $\frac{1}{2}(4x-1)^{\frac{1}{4}} + c$

o $-\frac{1}{3(8x+3)^{\frac{1}{4}}} + c$

p $\frac{3}{56}(7x-1)^{\frac{8}{3}} + c$

4

a $\frac{1}{3-x} + c$

b $-\frac{3}{2(x+1)^2} + c$

c $-\frac{1}{15(x-2)^3} + c$

d $-\frac{1}{3(x+8)^4} + c$

e $\frac{1}{9(1-3x)^3} + c$

f $\frac{9}{15-25x} + c$

g $-\frac{1}{24(2x-7)^4} + c$

h $-\frac{3}{10(x+4)^5} + c$

5 **a** $\frac{2}{3}(x+4)^{\frac{3}{2}}+c$

b $\frac{16}{3}(x-1)^{\frac{3}{2}}+c$

c $2\sqrt{x-1}+c$

d $12\sqrt{x+3}+c$

e $\frac{3}{4}(x+4)^{\frac{4}{3}}+c$

f $\frac{4}{7}(x-1)^{\frac{7}{4}}+c$

g $\frac{2}{5}(x+5)^{\frac{5}{2}}+c$

h $-\frac{2}{\sqrt{x-2}}+c$

i $\frac{32}{5}(x-6)^{\frac{5}{4}}+c$

j $\frac{5}{9}(x+1)^{\frac{6}{5}}+c$

k $\frac{15}{2}(x-6)^{\frac{2}{3}}+c$

l $\frac{24}{5}(x-1)^{\frac{1}{6}}+c$

6 **a** $\frac{2}{9}(3x+2)^{\frac{3}{2}}+c$

b $\frac{1}{10}(4x-1)^{\frac{5}{2}}+c$

c $\frac{15}{14}(2x-3)^{\frac{7}{5}}+c$

d $\frac{8}{7}(7x-1)^{\frac{1}{6}}+c$