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Functions - Lesson 2

Functions - Composition

LI

- Know what the composition of two functions is.
- Work out compositions of linear, quadratic, polynomial, trigonometric, exponential and logarithmic functions.

SC

- Algebra.

Jan 10-12:11

If the range of a function f is contained in the domain of a function g, then the outputs for f can be used as the inputs for g.

More precisely, if f is a function from A to B,

$$f : A \longrightarrow B$$

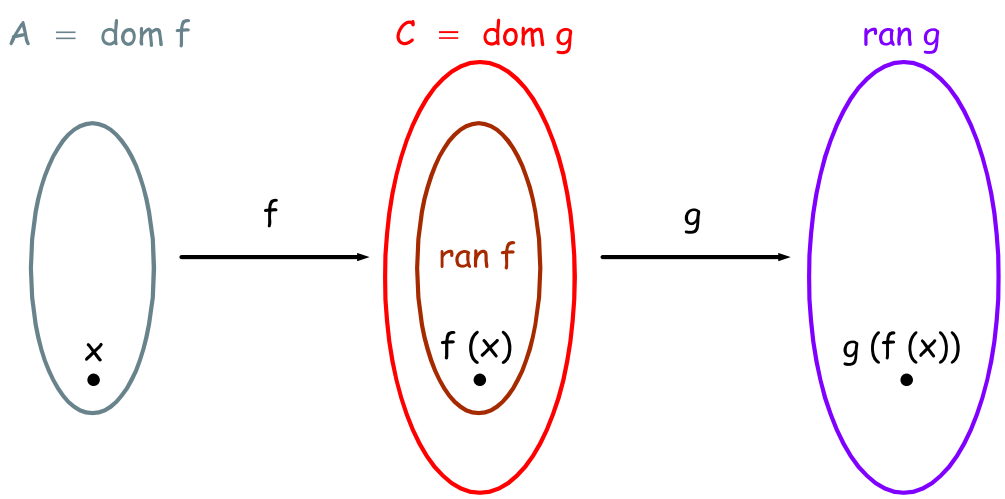
and g is a function from C to D,

$$g : C \longrightarrow D$$

and if ran f is contained within dom g ($= C$), then it makes sense to construct another function called the composition of g with f (denoted by $g \circ f$, and pronounced 'g circle f' or 'g of f') whose values in D are written as,

$$g(f(x))$$

We don't normally use $g \circ f$ in Higher Maths.



The composition of g with f is usually not the same as the composition of f with g

Example 1

If $f(x) = 2x + 9$ and $g(x) = x^2$, find :

(a) $f(g(x))$.

(b) $g(f(x))$.

(c) $f(f(x))$.

(d) $g(g(x))$.

(a) $f(g(x)) = f(x^2)$

$= 2(x^2) + 9$

$= \boxed{2x^2 + 9}$

(b) $g(f(x)) = g(2x + 9)$

$= \boxed{(2x + 9)^2 = 4x^2 + 36x + 81}$

(c) $f(f(x)) = f(2x + 9)$

$= 2(2x + 9) + 9$

$= 4x + 18 + 9$

$= \boxed{4x + 27}$

(d) $g(g(x)) = g(x^2)$

$= (x^2)^2$

$= \boxed{x^4}$

Sep 14-08:47

Example 2

If $f(x) = \dfrac{x}{1 - x}$, find $(f(f(x))$ as a fraction in its simplest form.

$$f(f(x)) = f\left(\frac{x}{1 - x}\right)$$

$$= \frac{\left(\frac{x}{1 - x}\right)}{1 - \left(\frac{x}{1 - x}\right)}$$

$$= \frac{x}{(1 - x) - x}$$

$$= \boxed{\frac{x}{1 - 2x}}$$

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Example 3

If $p(x) = \sin x$ and $n(x) = x^3$, find :

(a) $p(n(x))$.

(b) $n(p(x))$.

(c) $n(n(x))$.

(d) $p(p(x))$.

(a) $p(n(x)) = p(x^3)$

$= \boxed{\sin(x^3)}$

(b) $n(p(x)) = n(\sin x)$

$\boxed{\begin{aligned} &= (\sin x)^3 \\ &= \sin^3 x \end{aligned}}$

(c) $n(n(x)) = n(x^3)$

$= (x^3)^3$

$= \boxed{x^9}$

(d) $p(p(x)) = p(\sin x)$

$= \boxed{\sin(\sin x)}$

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Example 4

If $f(x) = \sqrt{x + 2}$ and $g(x) = 3 - x$, find $h(x) = f(g(x))$ and state a suitable domain for h.

$$\begin{aligned} h(x) &= f(g(x)) \\ &= f(3 - x) \\ &= \sqrt{(3 - x) + 2} \\ &= \boxed{\sqrt{5 - x}} \end{aligned}$$

We require $5 - x \geq 0$, i.e. we need $x \leq 5$. So,

$$\boxed{\text{dom } h = \{x \in \mathbb{R} : x \leq 5\}}$$

Example 5

If $D(x) = \log_3 x$ and $g(x) = x^2 + 4$, find $D(g(x))$ and $g(D(x))$.

$$D(g(x)) = D\left(x^2 + 4\right)$$

$$= \boxed{\log_3\left(x^2 + 4\right)}$$

$$g(D(x)) = g(\log_3 x)$$

$$= \boxed{(\log_3 x)^2 + 4}$$

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