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Applications of Calculus - Lesson 1

Maximum and Minimum Values on a Closed Interval

LI

- Determine the maximum and minimum values of a function on a closed interval.

SC

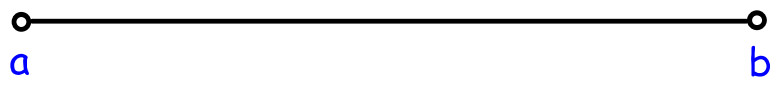
- Stationary points and values.
- Values of the function at the endpoints.

Jan 10-12:11

Intervals

An interval is a range of numbers, normally x-values

An open interval is an interval that does not include either endpoint

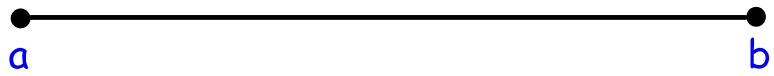


The white dots mean that the endpoints a and b are not included

The 'open interval from a to b' is written variously as,

$a < x < b$ or (a, b)

A closed interval is an interval that includes both endpoints



The black dots mean that the endpoints a and b are included

The 'closed interval from a to b' is written variously as,

$a \leq x \leq b$ or $[a, b]$

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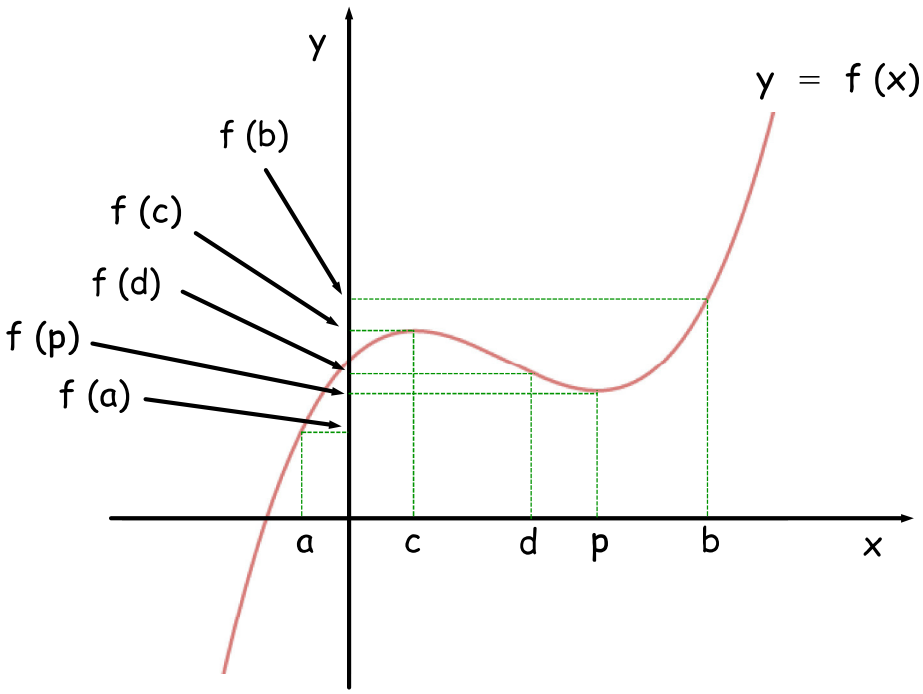
A function $y = f(x)$ on a closed interval $[a, b]$ will have a maximum value and a minimum value (meaning y - values)

(The above statement is not true for an open interval)



- On the closed interval $[a, b]$ the function has a maximum value of $f(b)$ and a minimum value of $f(a)$. Notice here that both the maximum and minimum values occur at the endpoints.

- On the closed interval $[d, b]$ the function has a maximum value of $f(b)$ and a minimum value of $f(p)$. Notice here that the maximum value occurs at an endpoint ($x = b$) and the minimum value occurs at a stationary point ($x = p$).

- On the open interval (a, b) the function does not have a maximum or minimum value, as the y-values will get arbitrarily close to $f(a)$ and $f(b)$, but never quite reach those values.

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Procedure for Finding Maximum and Minimum Values
of a Function on a Closed Interval [a, b]

- Find stationary points (x - and y - values). Remember that the y - values at stationary points are called stationary values.
- Find y - values at the endpoints, i.e. find f (a) and f (b).
- From the stationary values and f(a) and f (b), find the biggest and smallest y - values. These will be the required maximum and minimum values.

Example

Find the greatest and least values of the function,

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

on the interval $-4 \le x \le 11$.

Stationary Points

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

$$\therefore \quad f'(x) = x^2 - 4x - 12$$

For stationary points (SPs), $f'(x) = 0$:

$$x^2 - 4x - 12 = 0$$

$$\therefore \quad (x - 6)(x + 2) = 0$$

$$\Rightarrow \quad \underline{x = 6, x = -2}$$

$\underline{x = 6:}$

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

$$\therefore \quad f(6) = \frac{1}{3}(6)^3 - 2(6)^2 - 12(6) + 5$$

$$\Rightarrow \quad f(6) = 72 - 72 - 72 + 5$$

$$\Rightarrow \quad \underline{f(6) = -67}$$

$\underline{x = -2:}$

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

$$\therefore f(-2) = \frac{1}{3}(-2)^3 - 2(-2)^2 - 12(-2) + 5$$

$$\Rightarrow \quad f(-2) = -\frac{8}{3} - 8 + 24 + 5$$

$$\Rightarrow \quad \underline{f(-2) = \frac{55}{3}}$$

Mar 1-14:16

x	$\xrightarrow{-3}$	-2	$\xrightarrow{0}$	6	$\xrightarrow{7}$
$f'(x)$	$+$	0	$-$	0	$+$
Slope	/	—	\	—	/

$$f'(x) = (x - 6)(x + 2)$$

$$\therefore \quad f'(-3) = (-3 - 6)(-3 + 2)$$

$$\Rightarrow \quad \underline{f'(-3) = 9 > 0}$$

$$f'(x) = (x - 6)(x + 2)$$

$$\therefore \quad f'(0) = (0 - 6)(0 + 2)$$

$$\Rightarrow \quad \underline{f'(0) = -12 < 0}$$

$$f'(x) = (x - 6)(x + 2)$$

$$\therefore \quad f'(7) = (7 - 6)(7 + 2)$$

$$\Rightarrow \quad \underline{f'(7) = 9 > 0}$$

$\left(-2, \dfrac{55}{3}\right)$ is a local max,

and $(6, -67)$ is a local min.

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Endpoints

$x = -4$:

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

$$\therefore \quad f(-4) = \frac{1}{3}(-4)^3 - 2(-4)^2 - 12(-4) + 5$$

$$\Rightarrow \quad f(-4) = -\frac{64}{3} - 32 + 48 + 5$$

$$\Rightarrow \quad f(-4) = -\frac{1}{3}$$

$x = 11$:

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 12x + 5$$

$$\therefore \quad f(11) = \frac{1}{3}(11)^3 - 2(11)^2 - 12(11) + 5$$

$$\Rightarrow \quad f(11) = \frac{1\,331}{3} - 242 - 132 + 5$$

$$\Rightarrow \quad f(11) = \frac{224}{3}$$

Listing the relevant y - values gives,

$$f(6) = -67$$

$$f(-2) = \frac{55}{3} \approx 18.33$$

$$f(-4) = -\frac{1}{3} \approx -0.33$$

$$f(11) = \frac{224}{3} \approx 74.67$$

Maximum value of $f = \frac{224}{3}$

Minimum value of $f = -67$

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CfE Higher Maths

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