Integration - Lesson 1

Integrating Powers of a Variable
and Basic Rules for Integration

LI

- Know the meaning of Integration and Integral.

- Integrate functions of the form $f(x) = x^n$.

- Integrate a sum or difference of 2 or more functions.

SC

- Integration Rules.

- Rules of Indices.

- Rules of Fractions.

Differential Calculus

Integral Calculus is the study of calculating lengths, areas, volumes and other quantities by using differentials.

Integration is best thought of as the opposite process to differentiation.

The quantity obtained by integration is called an Integral.

Integrals are of 2 types :

- Indefinite. $\displaystyle\int$

- Definite. $\displaystyle\int_{a}^{b}$

Equivalent phrases for indefinite integrals :

Integral

Integrated function

Antiderivative

Antiderivative function

Primitive

Primitive function

Different Notations for Indefinite Integrals

If $y = f(x)$:

$\int y \; dx$ 'integral of y dx'

$\int f \; dx$ 'integral of f dx'

$\int y(x) \; dx$ 'integral of y of x dx'

$\int f(x) \; dx$ 'integral of f of x dx'

Integration Rules

Integrating a power of x

As $\dfrac{d}{dx} \dfrac{k}{n+1} x^{n+1} = k x^n$ (k and n are constants),

$$\int k x^n \, dx = \dfrac{k}{n+1} x^{n+1} + C$$

C is the integration constant
(aka constant of integration)

Integrating a sum or difference of functions

To integrate a sum/difference of several functions, integrate each function separately then add/subtract.

$$\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

<u>Example 1</u>

Find the integral of x^5.

$$\int x^5 \, dx = \frac{x^{5+1}}{5+1} + C$$

$$= \boxed{\frac{x^6}{6} + C}$$

Example 2

Integrate p^{-3} with respect to p.

$$\int p^{-3} \, dp = \frac{p^{-3+1}}{-3+1} + C$$

$$= \frac{p^{-2}}{-2} + C$$

$$\therefore \quad \boxed{\int p^{-3} \, dp = -\frac{p^{-2}}{2} + C \\ \left(= -\frac{1}{2p^{2}} + C \right)}$$

Example 3

Find the general antiderivative of $n^{3/4} + 6n^{-1/2}$.

Let $I = \int (n^{3/4} + 6n^{-1/2}) \, dn$.

$$\therefore \quad I = \int n^{3/4} + 6n^{-1/2} \, dn$$

$$= \int n^{3/4} \, dn + 6\int n^{-1/2} \, dn$$

$$= \frac{n^{3/4 + 1}}{3/4 + 1} + \frac{6n^{-1/2 + 1}}{-1/2 + 1} + C$$

Only need 1 constant;
the 2 constants C_1 and
C_2 can be added to give C

$$= \frac{n^{7/4}}{7/4} + \frac{6n^{1/2}}{1/2} + C$$

$$\therefore \quad I = \frac{4n^{7/4}}{7} + 12n^{1/2} + C$$

$$\left(= \frac{4\sqrt[4]{n^7}}{7} + 12\sqrt{n} + C \right)$$

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Questions

1   Integrate with respect to x.

   a x^6 **b** x **c** x^{-3} **d** 5 **e** $4x^3$

   f $-3x^{-7}$ **g** $\frac{3}{4}x^5$ **h** $8x^{-\frac{1}{4}}$ **i** $x^{\frac{3}{2}}$ **j** $x^{\frac{4}{3}}$

   k $x^{-\frac{1}{2}}$ **l** $-x^{-\frac{2}{5}}$ **m** $6x^{\frac{1}{2}}$ **n** $-\frac{1}{6}x^{-5}$

2   Integrate with respect to x.

   a $3x^2 + x - 1$ **b** $7 - 5x + x^4$ **c** $\frac{3}{2}x^5 - \frac{1}{4}x - 4$ **d** $\frac{4x^2}{3} - \frac{1}{5}x + 5x^{-6}$

3   Integrate with respect to x.

   a $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ **b** $10x^{\frac{3}{2}} - 21x^{\frac{5}{2}}$ **c** $\frac{2}{3}x^{\frac{2}{3}} - 4x^{-\frac{1}{5}} - 8x$ **d** $\frac{15}{4}x^{\frac{1}{4}} - \frac{2}{25}x^{-\frac{1}{5}}$

4   Integrate with respect to the given variable.

   a $k^4 + 3k - 5$ **b** $6p^8 - p^{-4}$ **c** $9t^{\frac{1}{2}} + 4t^{-\frac{1}{2}}$

Answers

1 **a** $\frac{x^7}{7} + c$

    b $\frac{x^2}{2} + c$

    c $-\frac{1}{2x^2} + c$

    d $5x + c$

    e $x^4 + c$

    f $\frac{1}{2x^6} + c$

    g $\frac{x^6}{8} + c$

    h $\frac{32x^{\frac{3}{4}}}{3} + c$

    i $\frac{2x^{\frac{5}{2}}}{5} + c$

    j $\frac{3x^{\frac{7}{3}}}{7} + c$

    k $2\sqrt{x} + c$

    l $-\frac{5x^{\frac{3}{5}}}{3} + c$

    m $4x^{\frac{3}{2}} + c$

    n $\frac{1}{24x^4} + c$

2 **a** $x^3 + \frac{x^2}{2} - x + c$

    b $\frac{x^5}{5} - \frac{5}{2}x^2 + 7x + c$

    c $\frac{x^6}{4} - \frac{x^2}{8} - 4x + c$

    d $\frac{4x^3}{9} - \frac{x^2}{10} - \frac{1}{x^5} + c$

3 **a** $\frac{2}{3}\sqrt{x}\,(x - 3) + c$

    b $2x^{\frac{5}{2}}(2 - 3x) + c$

    c $-4x^2 + \frac{2x^{\frac{5}{3}}}{5} - 5x^{\frac{4}{5}} + c$

    d $3x^{\frac{5}{4}} - \frac{x^{\frac{4}{5}}}{10} + c$

4 **a** $\frac{k^5}{5} + \frac{3k^2}{2} - 5k + c$

    b $\frac{2p^9}{3} + \frac{1}{3p^3} + c$

    c $2\sqrt{t}\,(3t + 4) + c$